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"Aircraft strength analysis"

Compendium of lectures

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The discipline "Aircraft strength analysis" is taught to students of the Faculty of Physics and Technology of the specialties "Design and Production of RKLA" and "Aircraft." The main basic textbook is “Building Mechanics of Flying Apparatuses” edited by I.F. Obraztsova. - M. Machine-building, 1986. - 536 s.

In this manual, the materials of the textbook in an abbreviated version and those that were not included in the specified textbook are given in part.

Subject 1 Basic definitions, aircraft integrity rate setting

Aircraft is a plane, launch vehicle or spaceship which has to take loads being functioned on it during operational process without damage and inadmissible shape change, which is to say being rigged and rugged enough. This requirement must be fulfilled by any engineering structure, but, aircraft structure has to be also differed by minimum mass requirement. That’s clear that the minimum mass requirement contradicts the requirements of having enough integrity and spring rate. Solving of this contradiction is one of the key problems arising under aircraft project development. It is carried out in calculations, design and experimental method process, both construction all the way around and its single parts and stipulates aircraft efficiency increasingly.

Aircraft strength analysis, like any kind of engineering analysis includes the following stages:

- Analytical model choice. A real – world object (aircraft) is studied and its characteristics being distinguished at that stage. It’s necessary to pass on from difficult construction to simple model.

- Analytical model analysis. Integrity and spring rate analysis of typical models by structural performance of materials and strength analysis methods.

- Backjumping from analytical model to real – world object and drawing conclusions towards its sufficient (unsufficient) integrity.

1. Analytical model choice.

Modern aircraft constructions are quite manifold. They consist of various elements which are differed by purpose of use, form, geometry, being used structural materials, methods of connecting them with each other, manufacture method etc.

Aircraft construction is divided into the variety of individual elements (accessories, tanks etc.) in strength analysis practice. Herewith, separate accessories interplay on each other with their gross productivity changes the activity of power factors in the form of axial and lateral forces, bending and rotational moments which are transferred through docking devices to connect the accessories.

That's why in aircraft strength analysis it's right to consider not just the constructions, but their analytical models which are simple models of real constructions with known theoretical solutions.

For the choice of right analytical model of real construction it's necessary to imagine yourself the assignment of every load – bearing element, pattern of its load and work feature of the whole construction.

First of all construction load-bearing element shape and size dimensions is carried out, which is to say the concept of load-bearing elements in a view of simple in shape elements (bar, beam, plate, panel, shell), for such ones are known the principles and methods of calculating in structural performance of materials.

Bar is a longish structural element that works on tensile and comprehension. One geometrical characteristic is really bigger than two others. The examples of bar model application in aircraft structure can be different elements of truss constructions and booms of the stiffened tanks.

Beam is a structural element that works on flection from lateral force efforts. As an example there may be some longish elements of device attachment. The wing can be considered as a clamped - free beam.

Panel is a flat or curved plate that takes up normal and tangent stresses. The panel is usually reinforced by axial and (or) lateral force sets. Flat and low arched panels are widely used in the analysis of wings structural elements, in analysis of dry compartments and attached tanks sheet work, in the structure elements of space aircraft main body.

Shell is the most complex element that takes up all kinds of loads. There are cylindrical, spherical, torus and conical shells, also attached and unattached ones.

Shells analytical models are used during propellant tanks, dry compartments, high – pressure bottles, air frames calculations.

The next stage of analytical model choice consists of analysis in load of structural element. The external loads that impacts on construction are also presented in simplified and comfortable form for calculation. There are surface and mass, distributed and concentrated, static and dynamic forces. Distributed loads are as a rule presented uniform or linear.

An important component of calculating model is a connection element diagram that is considered with another structural elements which preclude of its displacement under the influence of loads. There can be used hard fixation, hinge support, elastically pliable connection etc.

During the choice of analytical model, along with the catalogue it's required to consider the properties of the material from which construction elements are made of. Basic assumption during the material properties schematization consist in the fact that it is takin solid. It's necessary to have the functional connection between stresses and deformations for stresses and deformations in the construction elements (Hooke's Law or Plastic deformation Law). The material may be isotropic or anisotropic. Metals and its alloys are isotropic materials; Composite materials with directional reinforcement are orthogonal – anisotropic.

Thus, it's necessary to solve the idealization tasks during the analytical model choice:

- size dimensions and forms,
- grip conditions,
- loads,
- material properties.

Real construction schematization process leads to calculation mistakes. Taking the assumptions and simplifications it's necessary to imagine yourself the way they can influence on calculation results very clearly. It's necessary to follow the way like assumptions be taken only in material factor.

Strength analysis is a science about principles and methods of determining typical models deflected mode, its storage qualities analysis and dynamic behavior.

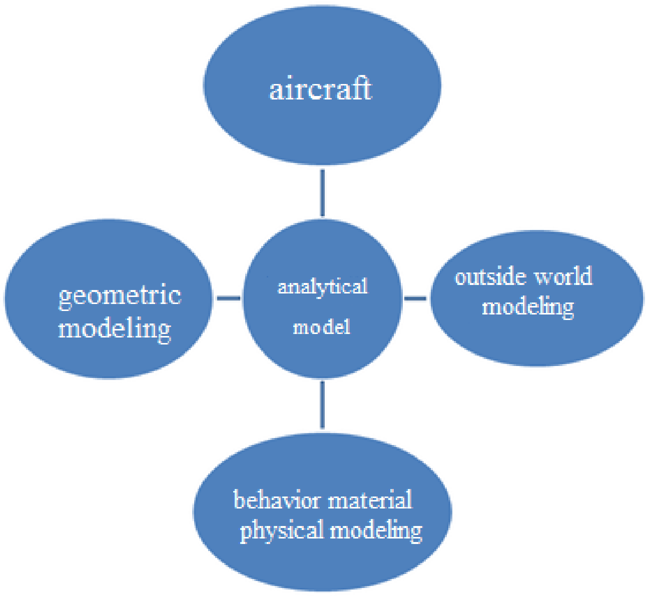
Aircraft strength analysis differs from other directions of this science mainly by thin structure analysis, and also by rising of requirement to calculating methods accuracy which considering construction mass limitation have to guarantee its safe work s material distinctive features.

Strength analysis **object of study** is space aircraft.

The aim is to learn the engineering methods of integrity, spring rate and rigidity aircraft calculating and their construction elements.

It's typical to use mathematical models for creating calculating engineering methods. **Aircraft mathematical model** consist of aircraft analytic model and calculating methods.

Aircraft analytical model is an aircraft simplified representation (abstraction). It is obtained due to efforts of aircraft geometric modeling, so the modeling of the outside world modeling and behavior material physical modeling.



Aircraft design – layout plan analysis is carried out for geometrical modeling performing. The following geometric primitives are used during the analysis: a bar, a plate, a shell and a massive body. It's essential to identify the construction elements structurally and to simulate them with primitives by analyzing the power flows. As an example, such construction elements as main body, aft end and tank simulate by force of shells and plates; boom, frame and connection elements is by force of bar; mating ring is by force of load cell and etc. **Decomposition structure** is carried out on this stage. The construction will consist of elements built by force of geometric primitives connected between themselves by means of inner forces. Inner forces come out like external loads.

That's why by performing the rocket layout analysis and item structure diagram analysis, the aircraft base structural decomposition is carried out. The construction is divided into the line of separate elements and compartments (The decomposition is under performing). Herewith, the separate components interplay on each other is changed by the acts of power, moments that are dispatched through the attachment fittings and mating rings. Aircraft design layout can structurally be submitted in a form of:

Element of space ship with aircraft	1. Frame, aft end, adapter module, low-drag fairing.	2. Boom, frame, longeron, frame structure.	3.Booms	3.Fasteners
Geometric model	Shell, plate	Bar	Load cell	Bolts, loft pins, cotter connections
Loading environments	Internal and external pressure, Axial force. Bending moment, Equated. Force.	Axial force Bending moment, Force.	Pressure, Axial force, Bending moment.	Axial force, Bending moment.

Outside world influence modeling: The outside world influenced on construction is being changed by the loads. They can be **earth-fixed** and **in-flight**. The main calculating ones is considered to be in-flight: operational in the instant of launch, load during the maximum air-velocity pressure, load during the staging, fairing ejection, in the case of cold vent. Each combination of these loads determines the simulation case.

Accelerations can be focused, surfacing and three dimensional. Besides, there also can be active and reactive, statistic and dynamic ones. Any variable of static load can be dynamic and can be determined by its **own vibration velocity**. If external load vibration velocity equals two or three periods of its own velocity interval, then such kind load is called dynamic. If it is more than four or five, then the varying duty is called static. Damage summation is emerging in this case that leads to appearing of construction micro fissures (fatigue). If the influence time is lower than acoustical wave transmission time, so it’s called shocked. The main construction element will be plate and shell in the static loading conditions during the strength analysis course. Physical modeling material behavior is used for construction reaction and behavior describing for environments. Each model is determined by the material properties (heating and power loadings). Strain, creep, fatigue strength of mechanism thermal mechanical area is used for models design.

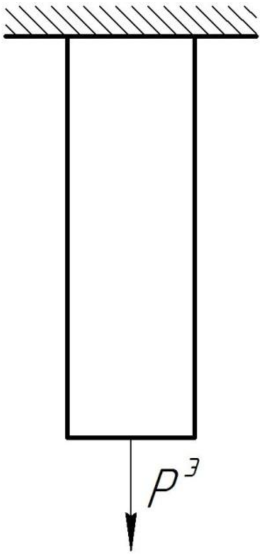
2. Analysis with factored design loads. Strength conditions.

Stress, deformations and displacement calculation in construction from intended external operations make up the basic strength analysis task.

Nevertheless, deformations and stresses analysis in aircraft construction elements is not still giving an availability to judge about their integrity and rigidity.

It’s required to have strength criterions that perform the balance between the integrity (stress, deformations, displacement etc.) balance in a form of chatter marks that determine the border between acceptable and unacceptable performances ranges.

In general mechanic engineering, for detail integrity, so having an availability of resisting failure, under that matter, the elastic theory is used the most widely. Let’s consider the example: a bar that is straining by «P». Strength condition:



$$\sigma^{e}_{max} = \frac{P^{e}}{F} \le [\sigma]$$

where P^e - a limit load, F is a cross-section area, $[\sigma]$ - an acceptable load, from which the detail integrity is provided. Acceptable load forms part from the boundary stresses. $[\sigma] = \frac{\sigma_{пред}}{n}$

$\sigma_{пред}$ – boundary stress. Depending on the load character and requirements to the detail there can be: integrity limit $\sigma_{в}$, liquid limit $\sigma_{т}$ etc. n – material factor. σ^{e}_{max} – a maximum stress in a dangerous detail crossing that is determined by the limit load.

Strength analysis for most parts of construction are performed with factored design loads in rocket and space technique that are accomplished by the way of limit load multiplication on some normalization factor which is called the material factor and, in most cases, it is marks like «f».

Under the material factor is understood the number that is more than one and what kind of it you should multiply the limit load level in order to get the ultimate load.

Limit load is received due to results of ballistic, aerodynamic and other analysis. Thus, construction is designed to operation of increased loads by means of material factor appliance. This loads increasing results to balance of:

- loads definition and lifting power inaccuracy by experimental and calculating way;
- design parameters acceleration and material mechanical characteristics;
- design models and methods proximity;
- possible deviations in aircraft tabs and tanks manufacture technology;
- possible deviations in operation conditions.

By all means, it's not profitable to create the construction with big material factor, because they will have huge mass. At this time, structural breakup during the decreasing of material factor is possible. During the material factor definition it's necessary to consider:

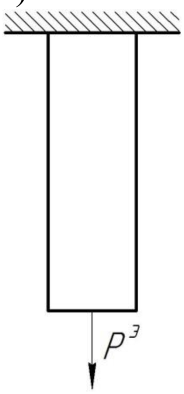
- construction layout;
- operating safe and unfailing performance necessary level;
- loading character;
- manufacturing process.

On this time to launch vehicle in flight conditions it's taken f = 1,25...1,5; during ground running – f = 1,5...2(2,25).

Material factors are defined of running experience in accordance with strength strength standarts.

Special document is developed for each aircraft class. It's called strength standarts that regiment safe coefficient value.

Getting back to an example:
Let's suppose the failure to be connected with bar explosion.
a)



Allowable stress design:

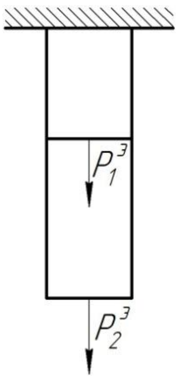
$$\sigma^e_{max} = \frac{P^e}{F} \le [\sigma]$$

$$\sigma^e_{max} = \frac{P^e}{F} \le \frac{\sigma_в}{n} \to F \ge \frac{P^e}{\sigma_в / n}$$

Factored stress design:

$$\sigma^p_{max} = \frac{P^p}{F} \le \sigma_в \qquad P^p = P^e f \to F \ge \frac{P^e f}{\sigma_в}$$

б)



$$\sigma^e_{max} = \frac{P^e_1 + P^e_2}{F} \le \frac{\sigma_в}{n} \to F \ge \frac{P^e_1 + P^e_2}{\sigma_в / n}$$

$$\sigma^p_{max} = \frac{P^e_1 f_1 + P^e_2 f_2}{F} \le \sigma_в \to F \ge \frac{P^e_1 f_1 + P^e_2 f_2}{\sigma_в}$$

Loads on aircraft that differ by nature are determined through different ways with various accuracy and probability of one or another load. That is why different loads make different f (It is settled by the strength standarts). Ultimate loads design allow to decrease the aircraft construction mass.

Boundary stresses.

Breakup is a loss of functional particulars, meaning the coming up in such condition when the construction stops satisfying to its application because of one or another kind of circumstances.

It's required to consider construction element strength conditions during the definition:

- its destination;
- load condition;
- demands that being taken for construction elements.

Strength assessment consist of comparing the equivalent analytical stresses defined by one or another failure theory, with integrity limit $\sigma_в$, with liquid limit $\sigma_{0,2}$ or endurance limit σ_{-1} depending from loading condition and demands to construction. If the compressive stresses appear in construction, so strength assessment on critical stresses is obligatory: $\sigma_{руйн} = \sigma_{кр}$.

Rigidness assessment is necessary for some construction elements, meaning the comparing of received moving values or deformations with boundary, which are defined according to field experience or another demands in such case. For example, there are facilities that can be applied for space aircraft to which adjustment accuracy demands is defined. Provided that, it's necessary for the deformations of some construction elements not lead to inadmissible displacement of these tools $\sigma_{гранич}(E_{гранич})$.

Simulation cases

Various loads work as a rule at one time during aircraft operation. For example, there is some drawing bar, internal pressure, gravity loads in one or a few ways. By the reason of thing that each load can change in time by its own law, so for each time moment the combining of these loads will be different. Structural integrity has to be practical enough during any loads combining, that is why it is essential to choose the most non-destructive one of these combinations for strength analysis and test operation.

The aircraft integrity is guaranteed in all kinds of load cases by the way of analysis aircraft load test corresponding to simulation cases.

Commanding load principle is one of the most spreading ones of simulation cases determination. Simulation cases fit with time point when one of the loads has the maximum possible value.

This approach is appropriate for space aircraft analysis. But, there can be cases during the another manufactures analysis when the loads combination which of them will not have the biggest value is going to be the most dangerous. The construction has to be rugged during the whole time of operation. The list of load simulation case is defined by analyzing the experience of aircraft creation and their operation for every element (cell).

The sequence of aircraft constructions structural analysis.

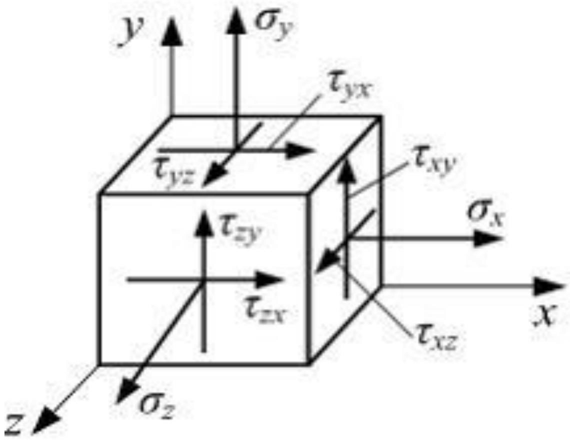
1. Layout and structure diagram learning.
2. Heating and load modes analysis.
3. Analysis of construction materials particulars in-service environment.
4. Design model choice for construction strength assessment.
5. Strain-stress state and construction lifting power calculation.
6. а) Design calculation (the determination of geometric characteristics of construction mail elements).

б) Checking calculation (safety coefficient determination, conclusion about having enough or not enough structural integrity).

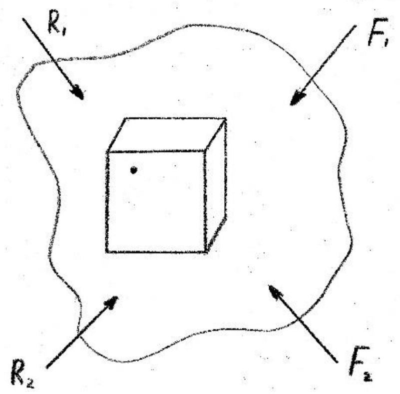
в) Lifting power definition (that boundary load which the element take without destruction).

Aircraft mathematical model creation is based on foundation of mechanics general approach of solid body deformation. This approach offers the formulation of static, geometric and physical construction formulas. These formulas blended decision lets us determine the strain-stress state. We should consider these equations.

Static equations:



Let's consider the construction that is situated in balance condition. Let's choose infinitely small element in a view of parallelepiped with dimensions dx, dy, dz in construction arbitrary point.



It’s as if dimensions and conditions are small and body forces operates on it in system of axes X, Y, Z. As all the construction is in balance, so the indivisible part is also in balance. Consider the balance condition of all forces sum that are headed towards x. Action stress is proportional at each area.

$$dV = dxdydz$$

$$\sigma_x dydz - \left(\sigma_x + \frac{\partial \sigma_x}{\partial x}dx\right)dydz + \tau_{zx}dxdy - \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}dz\right)dxdy + \tau_{yx}dxdz - \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}dy\right)dxdz + Xdxdydz = 0$$

We are having this during reduction:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} + \frac{\partial \tau_{yx}}{\partial y} + X = 0$$

Next, likewise, we are making the balance equation towards Y и Z, and getting three balance formulas:

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z} + Y = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + Z = 0$$

These differential equations are called static ones and are used during task solving of solid body deformation mechanics. Forming is happening during structural loading. It is determined in each point of displacement vector projection.

Coming into view: u, v, w.

$$\vec{r} = u\vec{i} + v\vec{j} + w\vec{k}$$

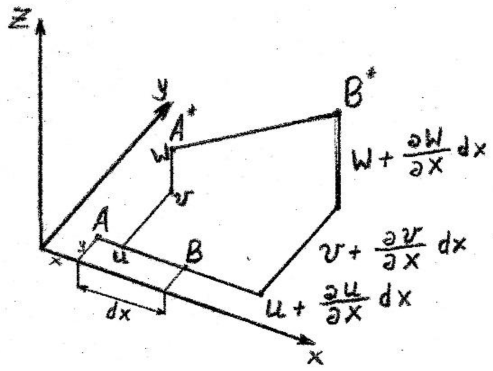
Besides, fractional variations of linear and corner deformations are coming between nearly points:

$$\varepsilon = \frac{A^*B^* - AB}{AB}$$

$$\gamma = \frac{\Delta\alpha}{\alpha}$$

Connection between displacement vector projection and relative strain is called **geometrical correlation.**

Geometrical correlation. Let’s determine the correspondence between the displacement vector projectors u, v, w and fractional deformations. Now, consider the interval AB with dimension dx that is in parallel with axis x.



Under the influence of load point A (x,y,z) will get the displacement projection u, v, w.

$$A^*(x+u; y+v; z+w)$$

$$B^*(x+dx+u+\frac{\partial u}{\partial x}dx; y+dy+v+\frac{\partial v}{\partial y}dy; z+dz+w+\frac{\partial w}{\partial z}dz)$$

$$A^*B^*=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$x_2-x_1=dx\left(1+\frac{\partial u}{\partial x}\right)$$

$y_2-y_1=\frac{\partial v}{\partial x}dx \qquad z_2-z_1=\frac{\partial w}{\partial z}dz$

$$A^*B^*=\sqrt{dx^2\left(1+\frac{\partial u}{\partial x}\right)^2+(\frac{\partial v}{\partial x})^2dx^2+(\frac{\partial w}{\partial z})^2dz^2} =$$

$$dx\sqrt{\left(1+\frac{\partial u}{\partial x}\right)^2+(\frac{\partial v}{\partial x})^2+(\frac{\partial w}{\partial z})^2} =$$

$$=\frac{dx\sqrt{\left(1+\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2}-dx}{dx}=$$

$$=\sqrt{1+2\frac{\partial u}{\partial x}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2}\text{-1}=(1+\alpha)^n=1+n\alpha+\cdots$$

$$\frac{\partial u}{\partial x};\frac{\partial v}{\partial x};\frac{\partial w}{\partial z}\ll 1$$

$$\alpha=2\frac{\partial u}{\partial x}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2$$

$$\varepsilon_x=1+\frac{1}{2}2\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2-1$$

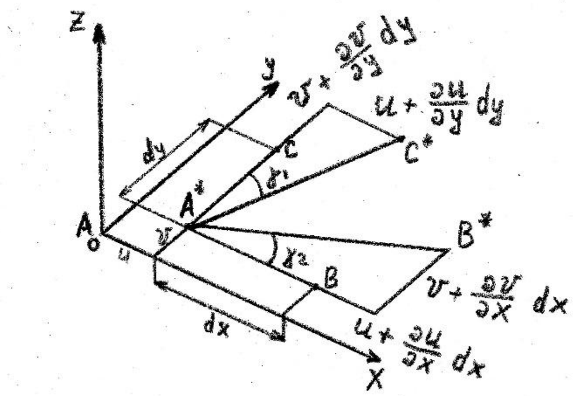
In the long run, we are getting the following formulas:

$$\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$
$$\varepsilon_z = \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

This general correlation is fairly for geometrically nonlinear tasks. Small deformations are being considered in the structural analysis course, then the second-order infinitesimal is being neglected, so we are getting:

$\varepsilon_x = \frac{\partial u}{\partial x}$ $\quad\quad$ $\varepsilon_y = \frac{\partial v}{\partial y}$ $\quad\quad$ $\varepsilon_z = \frac{\partial w}{\partial z}$



Let’s consider the angular variation of dimensions during the deformation.

$$\gamma = \gamma_1 + \gamma_2$$

$$tg\gamma_2 \approx \gamma_2 = \frac{v + \frac{\partial v}{\partial x}dx - v}{u + \frac{\partial u}{\partial x}dx - u + dx} =$$

$$= \frac{\frac{\partial v}{\partial x}dx}{\frac{\partial u}{\partial x}dx + dx} = \frac{\frac{\partial v}{\partial x}}{\frac{\partial u}{\partial x} + 1} \approx \frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} \ll 1$$

$$tg\gamma_1 \approx \gamma_1 = \frac{u + \frac{\partial u}{\partial y}dy - u}{v + \frac{\partial v}{\partial y}dy - v + dy} = \frac{\frac{\partial u}{\partial y}dy}{\frac{\partial v}{\partial y}dy + dy} = \frac{\partial y}{\partial y}$$

As the final results, we will get the following:

$$\gamma_{xy} = \frac{\partial y}{\partial y} + \frac{\partial v}{\partial x}$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

In general, the connection between u, v, w is defined by 6 geometrical equations:

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \frac{\partial y}{\partial y} + \frac{\partial v}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Each material has its own properties. We will be considering the material that possesses of uniformity, smoothness, perfect elasticity and isotropy in structural analysis course. The connections of material properties with stresses and deformation is defined by **physical formulas**.

Physical formulas

The connection between the stress and deformation is defined with the assistance of Hook's law formulas:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \mu(\sigma_y + \sigma_z))$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \mu(\sigma_x + \sigma_z))$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \mu(\sigma_y + \sigma_x)\right)$$

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

$$\tau_{zx} = G\gamma_{zx}$$

Where

$$\mu = \frac{\varepsilon_1}{\varepsilon}$$

$$G = \frac{E}{2(1+\mu)} - elesticity\ modulus\ of\ II\ type$$

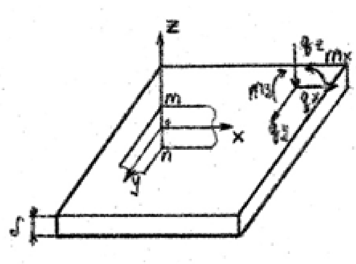
Quasi-static statement of thermal action consideration is used in case of temperature field, so the deformation will equal to zero of deformations forces.

The solving of any mechanics task of solid body deformation is being based on blended decision of 15 equations system and finding the 15 unknown ones: 6 physical equations, 6 geometrical and 3 static.

Plate theory

Plate is a body confined by two parallel planes the distance between ones is far smaller than the dimensions in the drawing.

Geometric multitude of points equidistant from subspace generator is called median surface.



Following equations should be solved for defining the stress state:

1. Static equations

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + x = 0 \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + y = 0 \\ \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + z = 0 \end{cases}$$

2. Geometric equations

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \\ \varepsilon_y = \frac{\partial v}{\partial y} \\ -\varepsilon_z = \frac{\partial w}{\partial z} \end{cases} \qquad \begin{cases} \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ -\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ -\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{cases}$$

3. Physical formulas

$$\begin{cases} \varepsilon_x = \frac{1}{E} \times (\sigma_x - \mu \times (\sigma_y + \sigma_z)) \\ \varepsilon_y = \frac{1}{E} \times (\sigma_y - \mu \times (\sigma_x + \sigma_z)) \\ \varepsilon_z = \frac{1}{E} \times (\sigma_z - \mu \times (\sigma_x + \sigma_y)) \end{cases} \qquad \begin{cases} \gamma_{xy} = \frac{\tau_{xy}}{G} \\ -\gamma_{yz} = \frac{\tau_{yz}}{G} \\ -\gamma_{zx} = \frac{\tau_{zx}}{G} \end{cases} \text{, где } G = \frac{E}{2 \times (\mu + 1)}$$

Suppositions are used for plates calculating:

- a normal interval mn with plate loading does not change its length..
- a normal interval mn before and after loading remains the normal one.
- Normal stresses in the plane that are parallel to median surface are being neglected.

These suppositions are called Kitchhoff hypothesis. They let us simplified the geometric, physical and static equations.

1) $\varepsilon_z = 0$ ⇨ $\varepsilon_z = \frac{\partial w}{\partial z} = 0$ ⇨ w=const ⇨ w=w(x;y) –deflection

2) $\gamma_{xz} = 0$ ⇨ $\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0$ ⇨ $\frac{\partial w}{\partial x} = \frac{\partial u}{\partial z}$ ⇨ $u(x;y) = u_0(x;y) - z \times \frac{\partial w}{\partial x}$

$$\gamma_{yz} = 0 \;\Rightarrow\; \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \Rightarrow \; \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y} \;\Rightarrow v(x;y) = v_0(x;y) - z \times \frac{\partial w}{\partial y}$$

The suppositions allow us to imagine the plate deformation process through the plate median surface deformation, through u_0, v_0 and deflection criterion w.

By dint of suppositions, the physical equations transmute into:

$$\begin{cases} \varepsilon_x = \frac{1}{E} \times (\sigma_x - \mu \times (\sigma_y + \sigma_z)) \\ \varepsilon_y = \frac{1}{E} \times (\sigma_y - \mu \times (\sigma_x + \sigma_z)) \\ \tau_{xy} = \frac{E}{2 \times (1-\mu^2)} \times (1-\mu) \times \gamma_{xy} \end{cases}$$

Now, we will apply the geometric formulas to the equations for stresses:

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z \times \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z \times \frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} - 2 \times z \times \frac{\partial^2 w}{\partial x \partial y} \end{cases}$$

$$\sigma_x = \frac{E}{(1-\mu^2)} \times \left(\frac{\partial u_0}{\partial x} - z \times \frac{\partial^2 w}{\partial x^2} + \mu \times \frac{\partial v_0}{\partial y} - \mu \times z \times \frac{\partial^2 w}{\partial y^2}\right)$$

$$= \frac{E}{(1-\mu^2)} \times \left(\varepsilon_{0x} + \mu\varepsilon_{0y} - z \times \; \times \frac{\partial^2 w}{\partial x^2} - \mu \times z \times \frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = \frac{E}{(1-\mu^2)} \times \left(\frac{\partial v_0}{\partial y} - z \times \frac{\partial^2 w}{\partial y^2} + \mu \times \frac{\partial u_0}{\partial x} - \mu \times z \times \frac{\partial^2 w}{\partial x^2}\right)$$

$$= \frac{E}{(1-\mu^2)} \times \left(\varepsilon_{0y} + \mu \times \varepsilon_{0x} - -z \times \frac{\partial^2 w}{\partial y^2} - \mu \times z \times \frac{\partial^2 w}{\partial x^2}\right)$$

$$\tau_{xy} = \varepsilon_{0xy} - 2 \times z \times \frac{\partial^2 w}{\partial x \partial y}$$

These applications show the deformation of any plate point and corresponding stresses can be evaluated through deformation and stresses in the median surface, and also coordinate values z, which defines the deformations and stresses criterions through bending and rotational moment criterions.

Let’s put in the criterions of line loads and moments for median surface:

$$N_x = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \sigma_x dz = \frac{E}{1-\mu^2} \times \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \left[\varepsilon_{0x} + \mu\varepsilon_{0y} - z \times \frac{\partial^2 w}{\partial x^2} - \mu \times z \times \frac{\partial^2 w}{\partial y^2}\right] \times dz$$

$$= \frac{E}{1-\mu^2} \times \left[(\varepsilon_{0x} + \mu\varepsilon_{0y}) \times z|_{-\frac{\delta}{2}}^{\frac{\delta}{2}} - \left(\frac{\partial^2 w}{\partial x^2} - \mu \times \frac{\partial^2 w}{\partial y^2}\right) \times \frac{z^2}{2}|_{-\frac{\delta}{2}}^{\frac{\delta}{2}}\right]$$

$$= \frac{E \times \delta}{(1-\mu^2)} \times (\varepsilon_{0x} + \mu\varepsilon_{0y})$$

Analogically N_y and N_{xy}.

$$M_x = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \sigma_x \times z \times dz = \frac{E}{1-\mu^2} \times \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \left[\varepsilon_{0x} + \mu\varepsilon_{0y} - z \times \frac{\partial^2 w}{\partial x^2} - \mu \times z \times \frac{\partial^2 w}{\partial y^2}\right] \times z \times dz = = \frac{E}{1-\mu^2} \times \left[(\varepsilon_{0x} + \mu\varepsilon_{0y}) \times \frac{z^2}{2}|_{-\frac{\delta}{2}}^{\frac{\delta}{2}} - \left(\frac{\partial^2 w}{\partial x^2} - \mu \times \frac{\partial^2 w}{\partial y^2}\right) \times \frac{z^3}{3}|_{-\frac{\delta}{2}}^{\frac{\delta}{2}}\right] = -\frac{E \times \delta^3}{12 \times (1-\mu^2)} \times \left(-\frac{\partial^2 w}{\partial x^2} + \mu \times \times \frac{\partial^2 w}{\partial y^2}\right)$$

Analogically M_y and M_{xy}.

Then, we are getting the criterions for line loads and moments for the median surface:

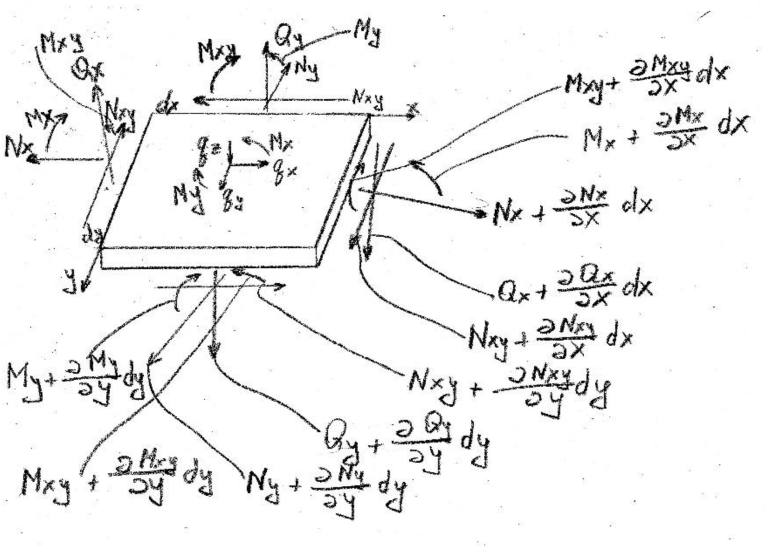
$$\begin{cases} N_x = \frac{E \times \delta}{(1-\mu^2)} \times (\varepsilon_{0x} + \mu\varepsilon_{0y}) \\ N_y = \frac{E \times \delta}{(1-\mu^2)} \times (\varepsilon_{0y} + \mu\varepsilon_{0x}) \\ N_{xy} = \frac{E \times \delta}{(1-\mu^2)} \times (1-\mu) \times \gamma_{xy} \\ M_x = -\frac{E \times \delta^3}{12 \times (1-\mu^2)} \times \left(-\frac{\partial^2 w}{\partial x^2} + \mu \times \frac{\partial^2 w}{\partial y^2}\right) \\ M_y = -\frac{E \times \delta^3}{12 \times (1-\mu^2)} \times \left(-\frac{\partial^2 w}{\partial y^2} + \mu \times \frac{\partial^2 w}{\partial x^2}\right) \\ M_{xy} = -\frac{E \times \delta^3}{12 \times (1-\mu^2)} \times (1-\mu) \times \frac{1}{2} \times \frac{\partial^2 w}{\partial x \partial y} \end{cases}$$

These equations show that line loads taking place in plate can be separated into two independent components: first three equations are a definition of median surface deformations, and the next three ones are deformations definition forced by bend.

$\frac{E \times \delta}{(1-\mu^2)} = B$ – compressive and extensional stiffness by median surface.

$\frac{E \times \delta^3}{12 \times (1-\mu^2)} = D$ – bending stiffness.

This application lets us consider the static side of task solving in a view of median surface infinitesimal area balance with dimension $dx \times dy$.



Now, we are getting all the strengths forced in the direction of Ox:

$$\sum x := -N_x dy - N_{xy} dx + N_{xy} dx + \frac{\partial N_{xy}}{\partial y} dxdy + N_x dy + \frac{\partial N_x}{\partial x} dxdy + q_x dxdy = 0$$

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + q_x = 0$$

To Oy: $$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + q_y = 0$$

To Oz: $$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q_z = 0$$

So, analogically, the all moments sum:

To Ox: $$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x + M_x = 0$$

To Oy: $$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y + M_y = 0$$

Static equations that define the plate internal forces are determined by 5 formulas which are mentioned above. Problem solving in the plate strain-stress state in defined by 17 formulas. This system can conditionally be imagined in a view of two independent loadings sum.

First loading corresponds to median surface deformation (tensile, compressive, shear). The second one is to median surface shear. This system is divided into the systems of 8 and 9 formulas.

$$\begin{cases} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + q_x = 0 \\ \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + q_y = 0 \\ N_x = B \times (\varepsilon_{0x} + \mu\varepsilon_{0y}) \\ N_y = B \times (\varepsilon_{0y} + \mu\varepsilon_{0x}) \\ N_{xy} = \frac{B\times(1-\mu)}{2} \times \gamma_{xy} \\ \varepsilon_{0x} = \frac{\partial u_0}{\partial x} \\ \varepsilon_{0y} = \frac{\partial v_0}{\partial y} \\ \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{cases} \text{ – tensile, compressive, shear.}$$

$$\begin{cases} \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x + M_x = 0 \\ \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y + M_y = 0 \\ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q_z = 0 \\ M_x = D \times (\chi_x + \mu \times \chi_y) \\ M_y = D \times (\chi_y + \mu \times \chi_x) \\ M_{xy} = \frac{D\times(1-\mu)}{2} \times \chi_{xy} \\ \chi_x = -\frac{\partial^2 w}{\partial x^2} \\ \chi_y = -\frac{\partial^2 w}{\partial y^2} \\ \chi_{xy} = -2 \times \frac{\partial^2 w}{\partial x \partial y} \end{cases} \text{ – bend}$$

.

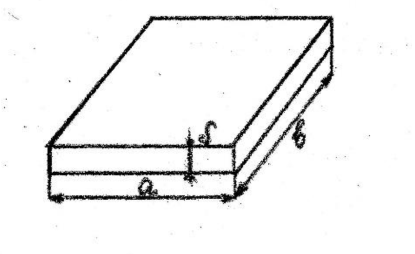
These mathematics allow us to define the stresses through inner forces:

$$\begin{cases} \sigma_x(z) = \frac{N_x}{\delta} + \frac{12 \times M_x}{\delta^3} \times z \\ \sigma_y(z) = \frac{N_y}{\delta} + \frac{12 \times M_y}{\delta^3} \times z \\ \tau_{xy}(z) = \frac{N_{xy}}{\delta} + \frac{12 \times M_{xy}}{\delta^3} \times z \end{cases}$$ – stresses through line loads.

The received arrangements show:

1) plate tensile, compressive, shear leads to median surface deformation;
2) deformation at any point of plate is defined through median surface deformation u_0, v_0 and crowning value. All points of plate are situated on median surface normal and have the same droop;
3) all plate layers paralleled to median surface are situated in the plane stress state condition;
4) stresses called by bend, in the point of median surface normal are changed according to linear law.

This theory is fair and has the experimental evidence for thickness attitude to the smallest plan size:

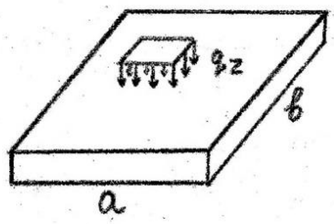


$$\frac{\delta}{a} \leq 0.2$$

If the behavior of droop to thickness $\frac{w}{\delta} < 0.5$, so the bending moments are dominating, and $\sigma_{bending}$ gives the linear dependence, but $\sigma_{tensile} \approx 0$. If $\frac{w}{\delta} > 0.5$, so this is membrane: $\sigma_{bending} \approx 0$, and $\sigma_{tensile} = const$.

Rectangular plate bend. Lagrange's equation.

Let's consider the rectangular plate with dimensions $a \times b$, thickness δ, that is situated under effect of line load q_z. The plate is fastened and is situated in the balance conditions.



It is required to solve the system including 9 formulas with 9 indeterminate ones for the plate stress state defining. Static, physical and geometric equations are entered into this system.

$$\begin{cases} \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x + M_x = 0 \\ \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y + M_y = 0 \\ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q_z = 0 \\ M_x = D \times (\chi_x + \mu \times \chi_y) \\ M_y = D \times (\chi_y + \mu \times \chi_x) \\ M_{xy} = \frac{D \times (1-\mu)}{2} \times \chi_{xy} \\ \chi_x = -\frac{\partial^2 w}{\partial x^2} \\ \chi_y = -\frac{\partial^2 w}{\partial y^2} \\ \chi_{xy} = -2 \times \frac{\partial^2 w}{\partial x \partial y} \end{cases}$$

χ_x, χ_y – the changing of median surface crookedness.

χ_{xy} – median surface torsion.

Now, let's solve this system.

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \quad \rightarrow \quad \frac{\partial Q_x}{\partial x} = \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y}$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \quad \rightarrow \quad \frac{\partial Q_y}{\partial y} = \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \times \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q_z = 0$$

$$M_x = D \times \left(-\frac{\partial^2 w}{\partial x^2} + \mu \times \frac{\partial^2 w}{\partial y^2}\right)$$

$$\frac{\partial^2 M_x}{\partial x^2} = D \times \left(-\frac{\partial^4 w}{\partial x^4} + \mu \times \frac{\partial^4 w}{\partial y^2 \partial x^2}\right)$$

$$M_y = D \times \left(-\frac{\partial^2 w}{\partial y^2} - \mu \times \frac{\partial^2 w}{\partial x^2}\right)$$

$$\frac{\partial^2 M_y}{\partial y^2} = D \times \left(-\frac{\partial^4 w}{\partial y^4} - \mu \times \frac{\partial^4 w}{\partial x^2 \partial y^2}\right)$$

$$M_{xy} = D \times (1 - \mu) \times \left(-\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$\frac{\partial^2 M_{xy}}{\partial x \partial y} = D \times (1 - \mu) \times \left(-\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)$$

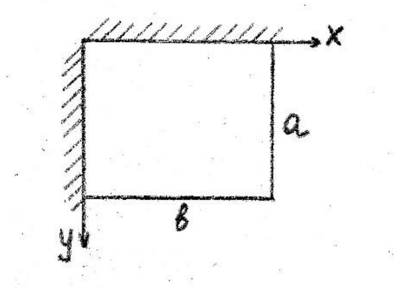
$$D \times \left(-\frac{\partial^4 w}{\partial x^4} + \mu \times \frac{\partial^4 w}{\partial y^2 \partial x^2}\right) + 2 \times D \times (1 - \mu) \times \left(-\frac{\partial^4 w}{\partial x^2 \partial y^2}\right) + D \times \left(-\frac{\partial^4 w}{\partial y^4} - \mu \times\right.$$
$$\left. \times \frac{\partial^4 w}{\partial x^2 \partial y^2}\right) + q_z = 0$$

$$D \times \left[-\frac{\partial^4 w}{\partial x^4} + \mu \times \frac{\partial^4 w}{\partial y^2 \partial x^2} - 2 \times \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2 \times \mu \times \frac{\partial^4 w}{\partial x^2 \partial y^2} - \frac{\partial^4 w}{\partial y^4} - \mu \times \frac{\partial^4 w}{\partial x^2 \partial y^2}\right] + q_z = 0$$

$$\frac{\partial^4 w}{\partial x^4} + 2 \times \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_z}{D}$$ - Lagrange’s equation.

This equation determines the whole class of bend rectangular plate issues. The concrete settlement is defined by a view of specified boundary conditions:

1) Fixing. It is equivalent to the demand that the droop value and angular deflection in fixing is equal to zero:

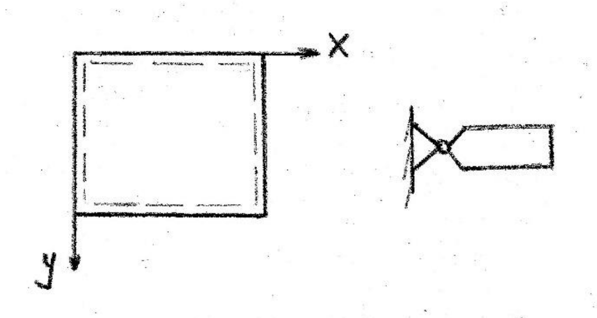


$$x = const \text{ и } w = 0 \rightarrow \frac{dw}{dx} = 0$$

$$y = const \text{ и } w = 0 \rightarrow \frac{dw}{dy} = 0$$

$$\begin{cases} \sigma_x = \frac{12 \times M_x}{\delta^3} \times z \\ \sigma_y = \frac{12 \times M_y}{\delta^3} \times z \\ \tau_{xy} = \frac{12 \times M_{xy}}{\delta^3} \times z \end{cases}$$

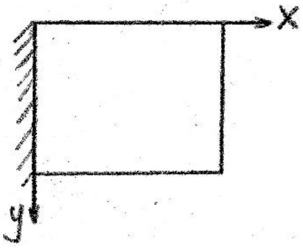
2) Hinged-fixed fixing.



$$x = const \text{ и } w = 0 \text{ и } M_x = 0 \rightarrow \frac{\partial^2 w}{\partial x^2} = 0$$

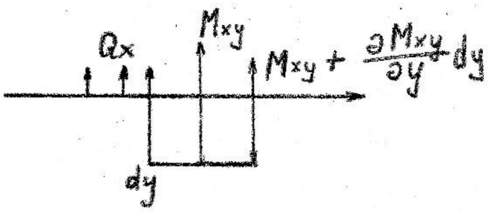
$y = const$ и $w = 0$ и $M_y = 0 \rightarrow \frac{\partial^2 w}{\partial y^2} = 0$

1) Plate free edge.



$$\begin{cases} \sigma_x = 0 \\ \sigma_y = 0 \\ \tau_{xy} = 0 \end{cases} \rightarrow \begin{cases} Q_x = 0 \\ M_x = 0 \\ M_{xy} = 0 \end{cases}$$

The rotational moment value M_{xy} is changed by spread load in a view of force couple during the plate free edge analyzing:



$Q^* = \frac{\partial M_{xy}}{\partial y}$

Let's change the rotational moment M_{xy} by force couples on a free edge that equivalent to adding shear force operating Q^*.

$x = const$ и $M_x = 0 \rightarrow \quad Q^{**} = Q_x + Q^*$
$y = const$ и $M_y = 0 \rightarrow \quad Q^{**} = Q_y + Q^*$

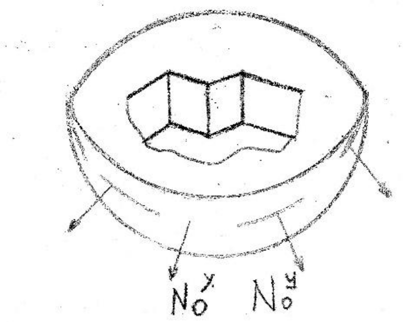
Calculation algorithm of plate stiffness task that works during the bend

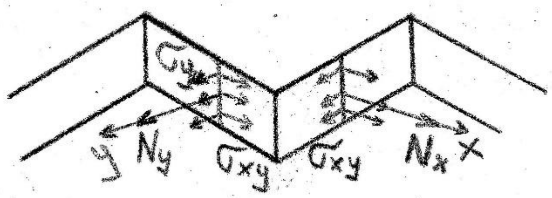
1. Lagrange's equation is solved with specified boundary conditions of plate fixing. Crowning value w is determined at any point of plate median surface.
2. Crookedness changing value and torsion median surface is defined according to known crowning w and geometrical formulas.
3. Inner line forces M_x, M_y, M_{xy} is defined according to median surface geometrics.
4. Normal and shearing stresses σ_x, σ_y, τ_{xy} are defined according to known inner forces at each point of plate.
5. Critical points are determined according to known stresses and inner forces. Equivalent stresses are defined with the assistance of strength theory for the critical

points and the functional capacity is being checked. Rigidity checking is carried out according to maximum deflection value w.

Plate planar loading

May it be some plate with the constant thickness h. The plane is stretched by known line loads N_0^x и N_0^y across the median surface. Let’s consider that there are not any external loads at all. Plate stress condition is defined as a result of blended decision of static, geometric and physical equations.





$$\begin{cases} \dfrac{\partial N_x}{\partial x} + \dfrac{\partial N_{xy}}{\partial y} = 0 \\ \dfrac{\partial N_y}{\partial y} + \dfrac{\partial N_{xy}}{\partial x} = 0 \end{cases}$$

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} \\ \varepsilon_y = \dfrac{\partial v}{\partial y} \\ \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{cases}$$

$$\begin{cases} \varepsilon_x = \dfrac{1}{E} \times (\sigma_x - \mu \times \sigma_y) \\ \varepsilon_y = \dfrac{1}{E} \times (\sigma_y - \mu \times \sigma_x) \\ \gamma_{xy} = \dfrac{E}{2 \times (1 + \mu)} \times \tau_{xy} \end{cases}$$

$$\begin{cases} \sigma_x = \frac{E \times h}{2 \times (1 - \mu^2)} \times (\varepsilon_x + \mu \times \varepsilon_y) \\ \sigma_y = \frac{E \times h}{2 \times (1 - \mu^2)} \times (\varepsilon_y + \mu \times \varepsilon_x) \\ \tau_{xy} = \frac{E \times h}{2 \times (1 + \mu)} \times \gamma_{xy} \end{cases} \qquad B = \frac{E \times h}{2 \times (1 - \mu^2)}$$

$$N_x = B \times h \times (\varepsilon_x + \mu \times \varepsilon_y)$$
$$N_y = B \times h \times (\varepsilon_y + \mu \times \varepsilon_x)$$

$$N_{xy} = \frac{B}{2} \times (1-\mu) \times h \times \gamma_{xy}$$

$$\varepsilon_x \to \frac{\partial^2}{\partial y^2} \qquad \frac{\partial^3 u}{\partial x \partial y^2} = \frac{\partial^2 \varepsilon_x}{\partial y^2}$$
$$\varepsilon_y \to \frac{\partial^2}{\partial x^2} \qquad \frac{\partial^3 v}{\partial y \partial x^2} = \frac{\partial^2 \varepsilon_y}{\partial x^2}$$
$$\gamma_{xy} \to \frac{\partial^2}{\partial x \partial y} \qquad \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial y \partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

Now, let's modify the last equations into the formulas for N_x, N_y and N_{xy}

$$\begin{cases} \varepsilon_x = \dfrac{1}{B \times h} \times (N_x - \mu \times N_y) \\ \varepsilon_y = \dfrac{1}{B \times h} \times (N_y - \mu \times N_x) \\ \gamma_{xy} = \dfrac{1}{B \times h \times (1-\mu)} \times 2 \times N_{xy} \end{cases}$$

$$\frac{\partial^2}{\partial y^2} \times \left(N_x - \mu \times N_y\right) + \frac{\partial^2}{\partial x^2} \times \left(N_y - \mu \times N_x\right) = 2 \times (1-\mu) \times \frac{\partial^2 N_{xy}}{\partial x \partial y}$$

Airy stress functions are used for solving of 3 equations with 3 indeterminate ones.

$$N_x = \frac{\partial^2 \varphi}{\partial y^2} \qquad N_y = \frac{\partial^2 \varphi}{\partial x^2} \qquad N_{xy} = \frac{\partial^2 \varphi}{\partial x \partial y}$$

The system of three formulas goes to 1 during Airy function substitution.

$$\frac{\partial^4 \varphi}{\partial y^4} + 2 \times \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi}{\partial x^4} = 0$$

The solving of this differential equation is based on specified boundary conditions. Airy functions are taken in a view of quadratic dependence of specified line loads N_0^x и N_0^y.

$$\varphi = \frac{1}{2} \times N_0^x \times y^2 + \frac{1}{2} \times N_0^y \times x^2 - N_0^{xy} \times x \times y$$

This law is good for flat plate and membranes.

$$u = \frac{x}{B \times h} \times (N_0^x - \mu \times N_0^y)$$

Line loads N_x, N_y и N_{xy} are defined for construction strength analysis through Airy functions, and then the stresses will be:

$$\begin{cases} \sigma_x = \frac{N_0^x}{h} \\ \sigma_y = \frac{N_0^y}{h} \\ \tau_{xy} = \frac{N_0^{xy}}{h} \end{cases}$$

The found line loads are substitute in geometric formulas and take integral for deformations defining, and then, we get:

$$u = \frac{x}{B \times h} \times (N_0^x - \mu \times N_0^y) + \frac{1}{B \times h} \times \frac{N_0^{xy}}{(1-\mu)} \times y$$
$$v = \frac{x}{B \times h} \times (N_0^y - \mu \times N_0^x) + \frac{1}{B \times h} \times \frac{N_0^{xy}}{(1-\mu)} \times x$$

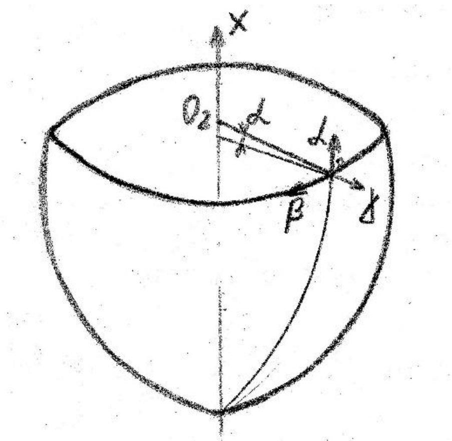
The found displacement fields let us define the placement of each point of flat plate that is situated in plane-stress state conditions. u and v are used for plate stiffness.

Circumgyration shell theory . General terms and defenitions

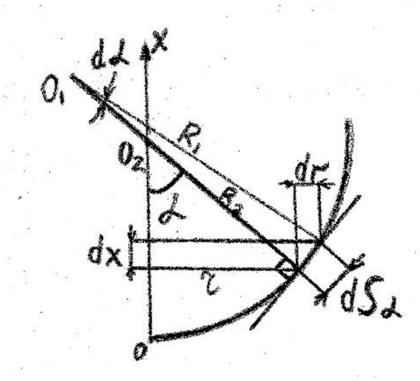
Rotational shell is risen as a result of moving line rotation towards the axis of symmetry.

Circumgyration shell theory. General terms and defenitions

Rotary shell is risen as a result of moving line rotation towards symmetry axis.



Geometrically, rotary shell can be defined force of 3-dimensional reference axis, which unit vectors are coincided with the moving line direction to circular meridians (parallels). Rotary shells are defined by thickness and curvature: R_1-moving line radius or meridian, R_2 – curvature radius circumferential direction or parallel. Curvature radios allow us to calculate any point of rotary shell. R_1 and R_2 are dependent quantities.



$$dS_\alpha = R_1 d\alpha$$

$$\sin\alpha = \frac{dx}{dS_\alpha}$$

$$\cos\alpha = \frac{dr}{dS_\alpha}$$

$$R_2 \sin\alpha = r$$

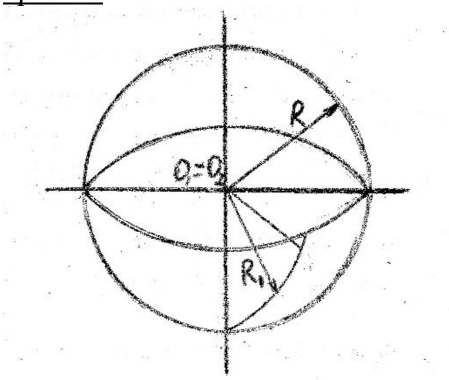
$$dr = \cos\alpha\, dS_\alpha = \cos R_1\, d\alpha$$

$$d(R_2 \sin\alpha) = R_1 \cos\alpha\, d\alpha$$

$$\frac{d}{d\alpha}(R_2 \sin\alpha) = R_1 \cos\alpha d\alpha$$

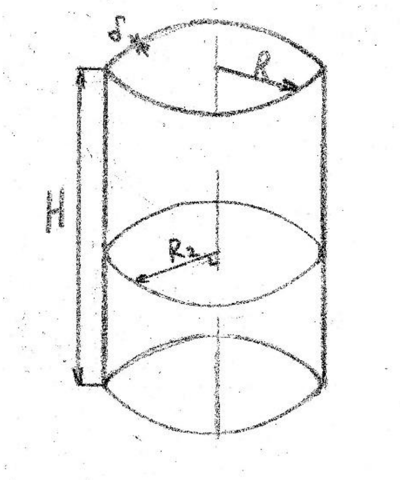
Let’s consider the real shells example:

Sphere:



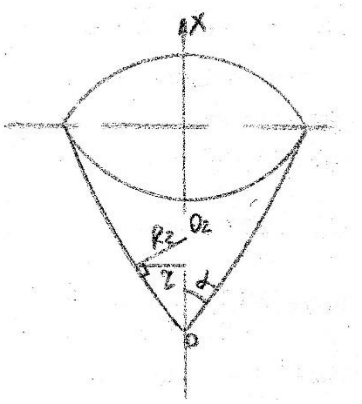
$R_1=R_2=R$

Cylinder:



R_2=$R_{cylinder}$=R , R_1=∞

__Cone:__



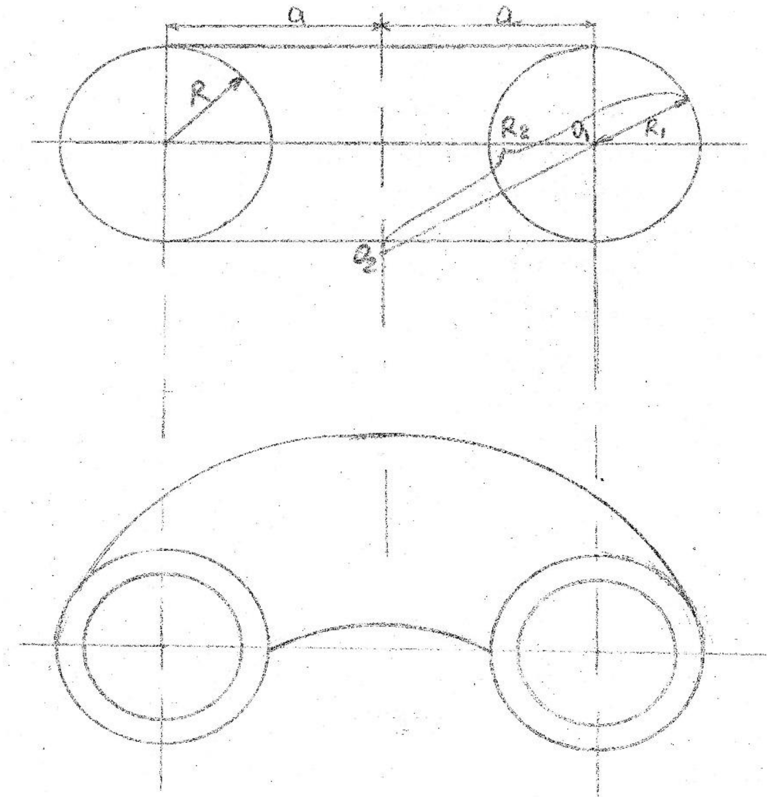
R_1=∞

$R_1 = \frac{r}{\cos\alpha}$

If the shell is more complex (ellipsoid):

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3}$$

__Torus shell:__



R_1=R

$$R_2 = R_1 + \frac{a}{\sin \alpha}$$

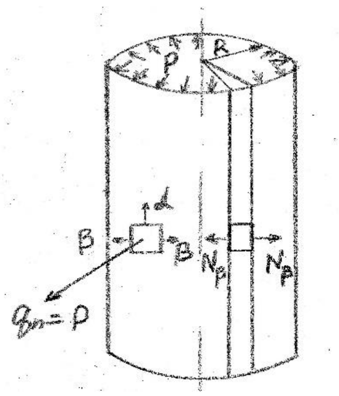
Shells strain-stress state analysis is carried out by force of simplified deformation. Allowances are used for this matter that are based on **Kirchhoff hypothesis:**

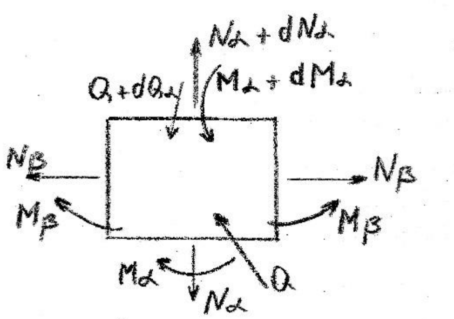
1. The normal line of a middle surface of a shell before and after loading remains the normal one.
2. The normal interval of a median surface doesn't change its length.
3. The normal stresses on the surfaces that parallel to the median surface are little and they are being neglected.

These allowances like in plate theory change the dimension of a problem into identify element (meaning the initial 3 – dimensional size turns into the two dimensional). Stress state can be imagined in a form of:

1. Momentless state;
2. The stress state called by bending moment.

<u>Let's consider the axially-symmetric loaded cylinder.</u>

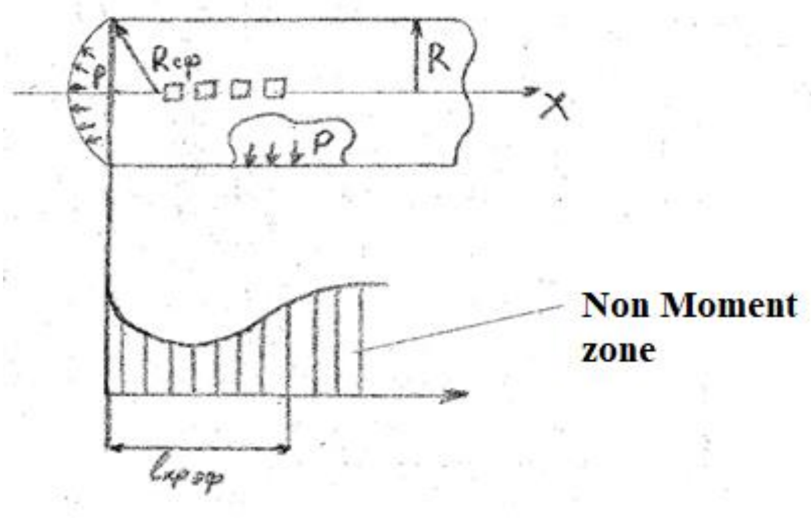




In case of the axially-symmetric loaded construction, shearing line axes $N_{\alpha\beta}$ and $M_{\alpha\beta}$ equal to zero, $N\beta$ and M_{β} don't depend on coordinate β, but they are constant. The stress can be described like a sum of zero-torque ones and with a moment one. The first stress is constant and the second one is varying during the zero-torque stress state. Moment state always takes place in the area of strengths and moments local action. Issues simplified solution is used during the structural design.

Now, we will consider the zero-torque stress state and determine the critical parameters of construction functional capability. In the general case a most definitive construction will be the one which don't have any strengths and moments localized concentrators, so this is a shell working in moment less state. The details are confirmed by experiment.

<u>Let's consider the cylindrical shell that has a dome head and being loaded by internal pressure.</u>

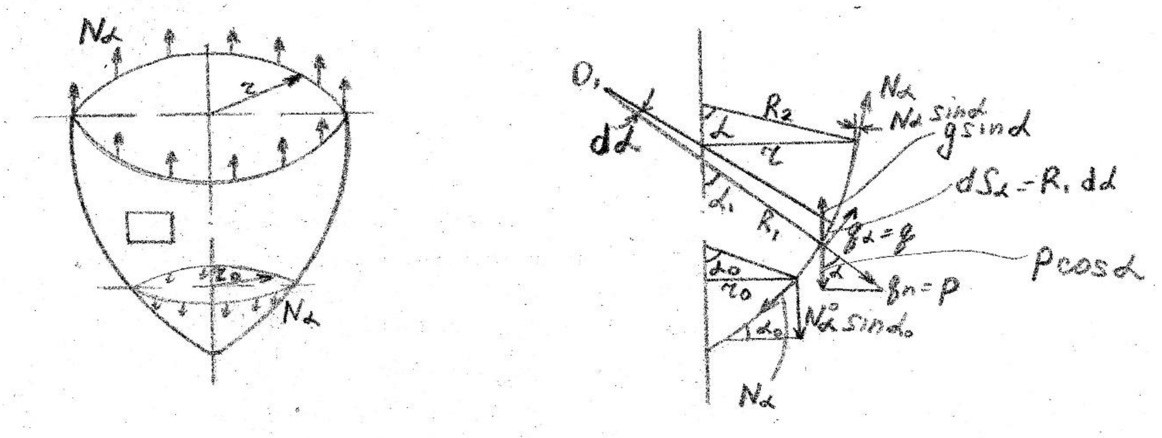


The moment state has an influence to minor parts of construction. The most of construction works zero-torque loading conditions and define through calculation is carried out after all about this, which takes into account the influence of bending moments from concentrated loads.

Let's consider the shell analysis with zero-torque theory.

Momentless shells theory. Static equations.

Let's consider the rotary shell loaded axially-symmetrical. In this case of loading, the deformation is happening that is defined by constancy rotationally not being depended on angle β. That means N_β doesn't depend on angle β.
Rotary shell in the balance conditions.



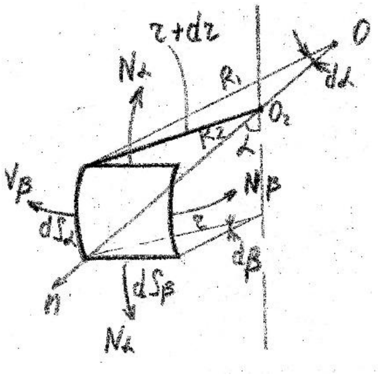
Now, write down the equilibrium criterion: the number of all strengths to axis x and set to zero.

$$\sum x: \; -2\pi r_0 N_{\alpha_0} \sin\alpha_0 + 2\pi r N_\alpha \sin\alpha + \int_{\alpha_0}^{\alpha} (\rho cos\alpha - q sin\alpha) 2\pi r d\alpha = 0$$

$$N_\alpha = \frac{1}{r sin\alpha}\left[-\int_{\alpha_0}^{\alpha} (\rho cos\alpha - q sin\alpha) 2\pi r d\alpha + r_0 N_{\alpha_0} sin\alpha_0\right]$$

If the shell is closed, so $N_{\alpha_0} = 0$ и $N_\alpha = \frac{1}{rsin\alpha}\int_{\alpha_0}^{\alpha}(\rho cos\alpha - qsin\alpha)2\pi r d\alpha$

Let's consider the second balance equation: the number of all strengths to a normal line and median surface is equal zero. Line loads will be appearing on the edges of this infinitesimal area.

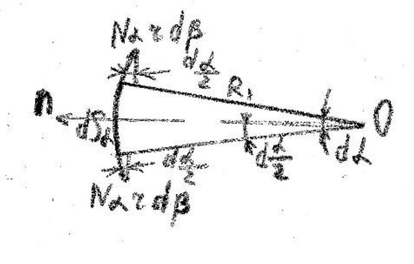


$$dS_2 = R_1 d\alpha$$
$$dS_\beta = rd\beta$$
$$r = R_2 \text{sin}\alpha$$

May this shell be loaded by internal pressing P. The thickness is constant and equals δ. Line loads will be appearing in the conditions of this symmetrical acceleration to the sides of infinitesimally small dS_α dS_β.

Axially symmetric load case defines that N_α and N_β doesn't depend on circumferential coordinate. Let's consider the forces that operate on lower and top edges for writing down the balance equations.

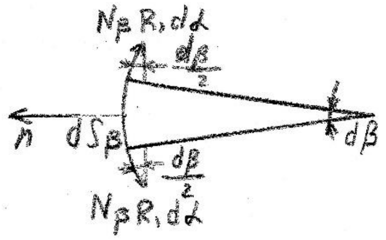
$$N_\alpha dS_\beta = N_\alpha r d\beta$$
$$(N_\alpha + dN_\alpha)(r + dr)d\beta = N_\alpha r d\beta + N_\alpha dr d\beta + dN_\alpha r d\beta + dN_\alpha dr d\beta = N_\alpha r d\beta$$



The resultant from the median line loads will equal:

$$N_\alpha r d\beta sin\frac{d\alpha}{2} + N_\alpha r d\beta sin\frac{d\alpha}{2} = 2N_\alpha r d\beta\frac{d\alpha}{2} = N_\alpha r d\beta d\alpha$$

Now, we are defining the resultant force operated on a directed line dS_α from line loads.



$$N_\beta dS_\alpha = N_\beta R_1 d_\alpha$$
$$N_\beta R_1 d_\alpha sin\frac{d\beta}{2} = N_\beta R_1 d_\alpha \frac{d\beta}{2}$$

This resultant acts on both edges of infinitesimally small element, that's why the sum of all circumferential forces will equal $N_\beta R_1 d_\alpha d\beta$.
This force is situated under the angle α and called the projection, subsequently the resultant will equal $N_\beta \sin\alpha R_1 d_\alpha d\beta$.
The resultant from external pressure to a normal line of median surface is equal:

$$R = PdS_\alpha dS_\beta = PR_1R_2 \sin\alpha\, d_\alpha d\beta$$

$\sum \bar{n}: -N_\alpha r d\alpha d\beta - N_\beta \sin\alpha R_1 d_\alpha d\beta + PR_1R_2 \sin\alpha\, d_\alpha d\beta = 0$

$N_\alpha R_2 \sin\alpha d\alpha d\beta + N_\beta R_1 d_\alpha d\beta \sin\alpha = PR_1R_2 \sin\alpha\, d_\alpha d\beta$

Laplace's equation: $\frac{N_\alpha}{R_1} + \frac{N_\beta}{R_2} = P$

Static equations let us determine meridian and circular line loads for general shell. Line loads call the constant the constant about thickness stresses. The values of these stresses equal:

$$\sigma_\alpha = \frac{N_\alpha}{\delta};\ \sigma_\beta = \frac{N_\beta}{\delta}$$

The arising stresses conform to the arising deformations that are determined by Hooke law formula:

$$\varepsilon_\alpha = \frac{1}{E}(\sigma_\alpha - \mu\sigma_\beta)$$

$$\varepsilon_\beta = \frac{1}{E}(\sigma_\beta - \mu\sigma_\alpha)$$

It's necessary to define the deformation during the stiffness task. Relative deformation value is evaluated through the interchange vector components in a case of axially symmetric loading in the following manner:

$$\varepsilon_\alpha = \frac{1}{R_1}\left(\frac{du}{d\alpha} + \frac{w}{R}\right)$$
$$\varepsilon_\beta = \frac{1}{r}(u\cos\alpha + w\sin\alpha)$$

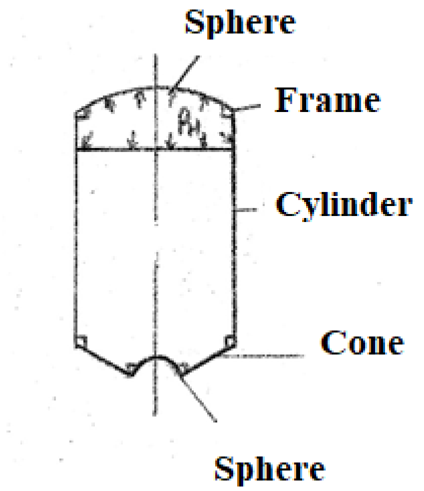
where u and w – circular and radial movement accordingly.

Calculation algorithm of axially symmetric shells stressing is the following:

1. Line loads N_α and N_β are found through the static equations.

2. Stresses are defined according to line loads.
3. Failure zone is determined by analyzing the stress fields. Stresses in these zones are used for carrying out of checking calculation, design calculation, strength calculation and additional loads calculation.
4. Relative deformations ε_α and ε_β and also movement vector projection are defined according to stresses in the case if stiffness limitations for shell are assigned.

The presented algorithm is used after the analyzing element decomposing as the aircraft construction has a difficult geometry. Algorithm will be the following (example: propellant tank):

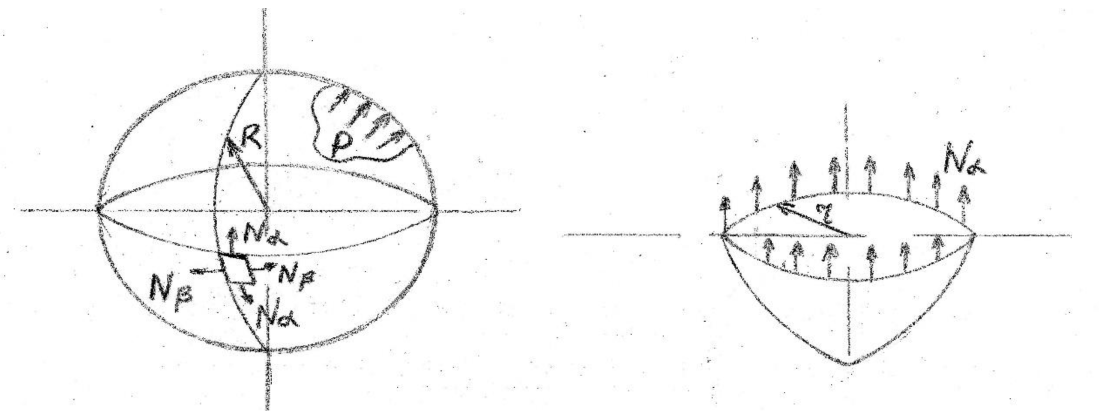


1. Now, you are accomplishing the simplifying or decomposition.
2. Then, you are determining the loading simulation case.
3. Then, you are caring out the strength analysis of every shell element for each simulation case.
4. Then, you are determining the construction geometry and all elements total weight from the strength condition.
5. Then, you are caring out the stress state alignment in the edge effects zone. In this case, you are solving the moment task for every shell and define the stress fields. After this, you are caring out the performance evaluation and geometry healing if it's necessary in the construction concentrator area on the stress fields.
6. Then, after the specified calculations the documentation for a tab or an element is being produced. The specimen is being manufactured and the tests are being performed according to this.

Let's consider the particular cases of shells loaded by internal pressing for problem solving:

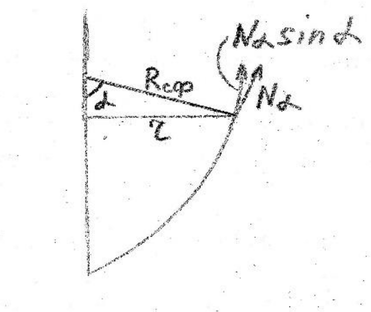
<u>*Object 1.*</u>

Spherical shell calculation loaded with internal pressing. The shell has a constant thickness.



$$R_{sphere} = R_1 = R_2$$

If we use the static equation for finding the internal line loads, we are getting:



$$2\pi r \times N_\alpha \sin\alpha = \pi r^2 \rho$$
$$r = R_{sphere} \times \sin\alpha$$
$$R_{sphere} \times 2\pi \times N_\alpha \sin^2\alpha = {R_{sphere}}^2 \times \sin^2\alpha\, \rho\pi$$
$$N_\alpha = \frac{PR_{sphere}}{2}$$
$$N_\beta = \frac{Psphere}{2}$$

We are using the Laplace equation for defining of the second formula

$$\frac{N_\alpha}{R_1} + \frac{N_\beta}{R_2} = P$$
$$\frac{N_\alpha}{R_{sphere}} + \frac{N_\beta}{R_{sphere}} = P$$

Efforts amount let us find the stresses:

$$\sigma_\alpha = \frac{N_\alpha}{\delta}$$
$$\sigma_\beta = \frac{N_\beta}{\delta}$$

This formulation allows us to write down the strength conditions and on the ground of it we will be able to solve the check and design problems for spherical shells.

$$\sigma_\alpha = \frac{PR_{sphere}}{2\delta}$$
$$\sigma_\beta = \frac{PR_{sphere}}{2\delta}$$
$$\sigma_{эквI} = \frac{PR_{sphere}}{2\delta} \leq [\sigma]$$
$$\sigma_{эквIII} = \sigma_1 - \sigma_3 = \frac{PR_{sphere}}{2\delta} \leq [\sigma]$$

Structural analysis is always performed according to design limit loads $P^p = P_{экспл} \times f$
Material factor for the tanks f=1.5
For spherical tanks f=2.25
These common factors allow us to solve:

Check task:

$$\sigma_{эквIII} = \frac{P^p R_{sphere}}{2\delta} \leq [\sigma]$$

Design task presumes finding the shell thickness:

$$\delta \geq \frac{P^p R_{sphere}}{2[\sigma]}$$

Lifting power calculation is carried out in the following way:

$$P^P \leq \frac{2\delta[\sigma]}{R_{sphere}}$$

If there is a structural rigidity objective, so the length – diameter ratio ε_α and ε_β are being defined.

$$\varepsilon_\alpha = \frac{1}{E} \times (\sigma_\alpha - \mu\sigma_\beta)$$
$$\varepsilon_\beta = \frac{1}{E} \times (\sigma_\beta - \mu\sigma_\alpha)$$

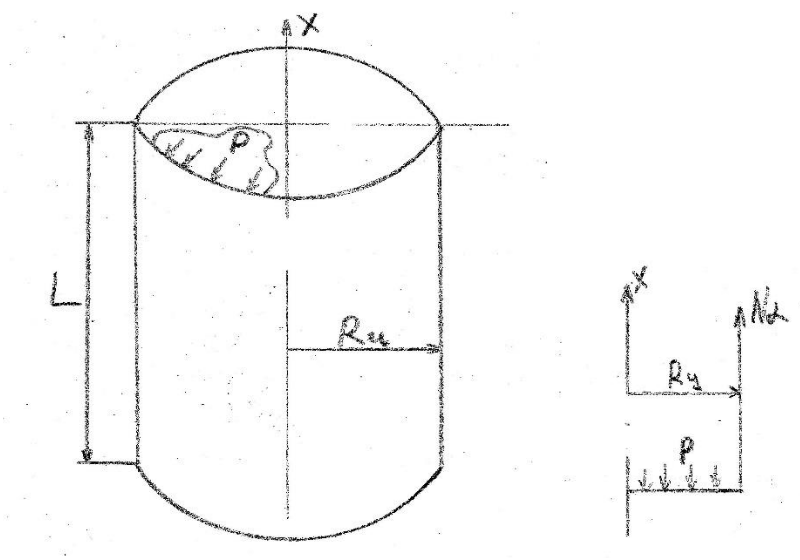
$\varepsilon_r = \frac{w}{R_{сф}}$ - radios direction, where

$$w \leq [w]$$

The characteristic property of the spherical shell is the fact that it is equi-loaded in all directions $\sigma_\alpha = \sigma_\beta$. The sphere will always have lower weight and lateral area than any other shell with specified tank volume.
Shell works on compression during this acceleration. Buckling phenomenon is possible. In this case, yield stress is the biggest loading that the shell can resist. Breaking point is the biggest stress during the behavior on tension.

Object 2.

Cylindrical shell calculation with the constant thickness loaded by internal pressure



$$R_1 = \infty$$
$$R_2 = R_{cyleinder}$$

Now, let’s determine the internal line loads according to statistics formula:

$$N_\alpha = \frac{PR_{cylinder}}{2}$$
$$N_\beta = PR_{cylinder}$$

These internal forces let us find the stresses:

$$\sigma_\alpha = \frac{N_\alpha}{\delta}$$
$$\sigma_\beta = \frac{N_\beta}{\delta}$$
$$\sigma_\alpha = \frac{PR_{cylinder}}{2\delta}$$
$$\sigma_\beta = \frac{PR_{cylinder}}{\delta}$$

We also take into account the fact that structure analysis is performed by ultimate loads $P^p = P_{эксп\text{л}} \times f$.
Obviously, this stress allows us to define the equivalent loads and solve the following tasks according to them.

Check task:

$$\sigma_{экв I} = \sigma_{max} = \sigma_\beta = \frac{P_{экс} f R_{cylinder}}{\delta} \leq [\sigma]$$

Design task:

$$\delta \geq \frac{P_{экс} f R_{cylinder}}{[\sigma]}$$

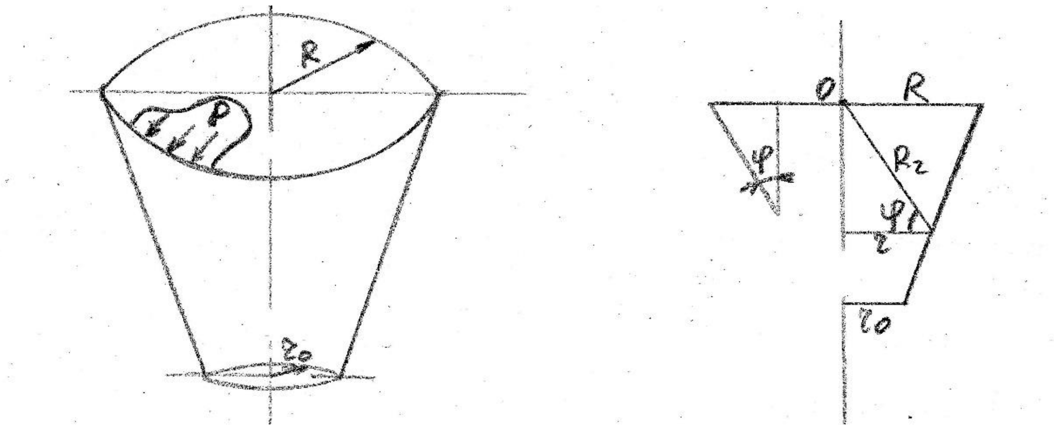
Lifting power calculation:

$$P^{\text{экс}} = \frac{\delta[\sigma]}{fR_{cylinder}}$$

We are acting analogically during **rigidity** task solving like in spherical one. The characteristic property of this shell is that the construction is not a full-strength one. There is a stress appearing in circular direction that is twice as big as medial stresses. It follows that operative cylinder won't have the minimum mass for pre-designed volume of cylindrical shell. Definitive cylindrical shells are received in the consequence of their waffle shell design. Composite materials also allow us to implement the full-strength cylindrical shell.

Object 3.

Cone shell calculation with the constant thickness loaded by its own pressing



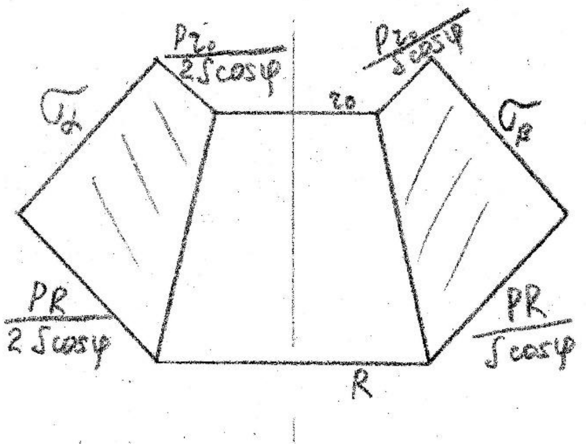
$$R_1 = \infty$$
$$R_2 = \frac{r}{\cos\varphi}$$

Now, let's determine the internal line loads:

$$N_\alpha = \frac{Pr}{2\cos\varphi}$$
$$N_\beta = \frac{Pr}{\cos\varphi}$$

Then, find the stresses:

$$\sigma_\alpha = \frac{N_\alpha}{\delta}$$
$$\sigma_\beta = \frac{N_\beta}{\delta}$$
$$\sigma_\alpha = \frac{Pr}{2\delta\cos\varphi}$$
$$\sigma_\beta = \frac{Pr}{\delta\cos\varphi}$$



This approach lets us solve the check and design tasks and also to calculate the allowable loads. Now, let's determine the operation loads by applying the safety factor for this.

Check task:

$$\sigma_{экв III} = \sigma_{max} - \sigma_{min} = \sigma_\beta - 0 = \frac{PR}{2\delta \cos\varphi} \leq [\sigma]$$

Construction is operative in case of:

$$n = \frac{[\sigma]}{\sigma_{экв III}} \geq 1$$

Design task:

$$\delta \geq \frac{PR}{2[\sigma]\cos\varphi}$$

Lifting power calculation:

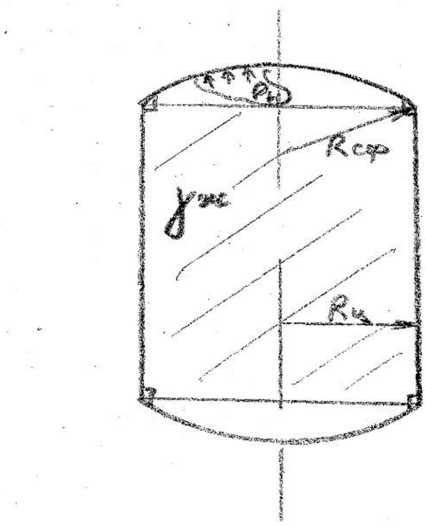
$$P^{max} = \frac{2\delta[\sigma]\cos\varphi}{R}$$

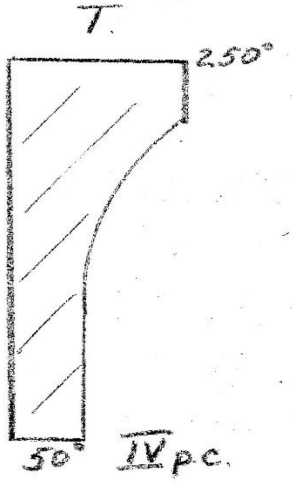
There is an inhomogeneous stress occurring in cone shell that is depend on cone half angle and reference radius.

Critical section is defined in the cone with the biggest radius. The amount of circular and median values differs by two times. It's recommended to use power stiffening in meridian and axis direction or to use the waffle shell for receiving of optimal structure.

Object 4.

Cylindrical tank calculation with bumped head

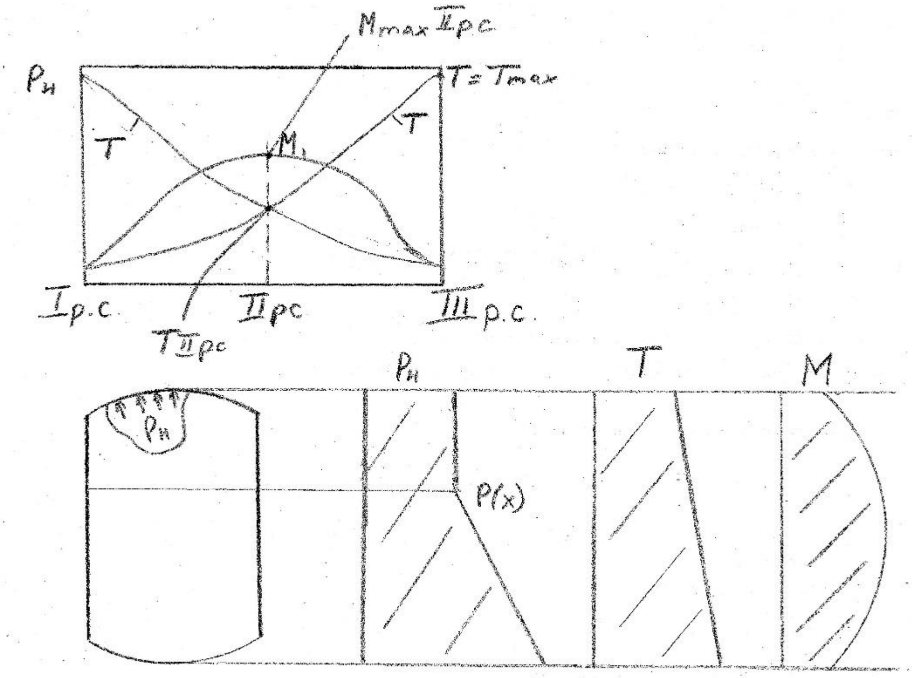




$$R_{\text{сф}} = 1{,}2 \div 1{,}3 R_{cylinder}$$
$$R_{\text{сф}} = 1{,}3 R_{cylinder}$$

This structure calculation is stipulated by loading condition. The loading conditions are reverse that's why the most important simulation cases are being separated. There are four loading simulation cases that are used for tank:

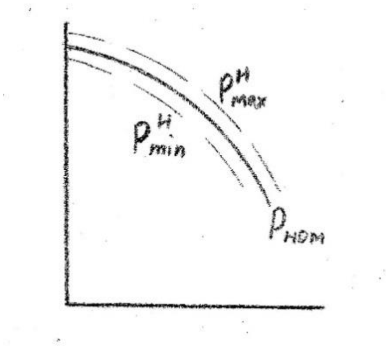
1. Maximal inflation situation;
2. Tank loading case by maximal bending moment and axial compressing force which is in evidence in max-q area. Altitudes from 8km to 12km conform to it.
3. Operations of maximal axial compressing force. This loading case conforms to stage separation area. Load factor capability and axial compressing force exert at this moment.
4. Temperature difference of structure heating from 250° in the area of high-temperature inflation and till 50°.



It’s necessary to make a certain structural analysis for each simulation case.

The decomposition is performed previously and then, we are getting structural members kit for which calculation for these simulation cases is carried out:

1. Integral tank. It means the aft end takes up the operating of axial force and bending moments. Overpressure is included that is involved with security valve operation during internal pressure loading.



$$\mathrm{P_н} = \mathrm{P_{ном\ мах}} + \Delta\mathrm{p}$$

where $\Delta\mathrm{p} = 0{,}1\text{МПа}$

In spherical shell arises:

$$\sigma_\alpha = \sigma_\beta = \frac{PR_{sphere}}{2\delta}$$

All imprecisions are considered during the calculations:

$$P^p = P_{\text{экспл}} \times f_{\text{р}}$$

where $f_{\text{р}} = 1{,}5$
These conformities let us solve the design task:

$$\delta \geq \frac{(\mathrm{P_{ном\ мах}} + \Delta\mathrm{p}) \times f_{\text{р}}\, R_{sphere}}{2[\sigma]}$$

Stresses take the value: $[\sigma] = \sigma_\beta$

This operation is also repeated for the lower aft end:

$$\mathrm{P_н} = (\mathrm{P_{ном\ мах}} + \Delta\mathrm{p}) + \gamma_{ж}\mathrm{H}$$

$$\delta \geq \frac{\big((\mathrm{P_{ном\ мах}} + \Delta\mathrm{p}) + \gamma_{ж} n_x \mathrm{H}\big) \times f_{\text{р}}\, R_{sphere}}{2[\sigma]}$$

where n_x is a load factor.

It is worth mentioning that mechanical-and-physical properties **depend on temperature** literally:

$$\delta \geq \frac{(\text{P}_{\text{ном мах}} + \Delta\text{p}) \times f_{\text{p}}\, R_{sphere}}{2[\sigma]_t}$$

Temperature effect is solved by breaking point calculation and elasticity modulus for temperature of structure operation in structure members. Variable of pressure height operates in cylinder.

$$\sigma_\alpha = \frac{(P_{\text{н}} + \Delta\text{p})\, f R_{\text{ц}}}{2\delta_{\text{ц}}} = [\sigma]_t$$

$$\sigma_\beta = \frac{(P_{\text{н}} + \Delta\text{p}) f R_{\text{ц}}}{\delta_{\text{ц}}} = [\sigma]_t$$

$$\sigma_{\text{экв}I} = \sigma_{max} = \sigma_\beta = \frac{(P_{\text{н}} + \Delta\text{p})\, f R_{\text{ц}}}{2\delta_{\text{ц}}} \leq [\sigma]_t$$

$$\sigma_\alpha = \frac{(P_{\text{н}} + \Delta\text{p})\, f R_{\text{ц}}}{2\delta_{\text{ц}}} \leq [\sigma]_t$$

$$\sigma_\beta = \frac{(P_{\text{н}} + \Delta\text{p}) f R_{\text{ц}}}{\delta_{\text{ц}}} \leq [\sigma]_t$$

$$\delta_{\text{ц}} \geq \frac{(P_{\text{н}} + \Delta\text{p}) f R_{\text{ц}}}{[\sigma]_t}$$

We are choosing the bigger thickness between two ones of the spherical shell, so this thickness is corresponding for the first simulation case. The second and the third calculation are performed in the following manner.

2. Combined action of axial force and bending moment carry in the conception of equivalent force $T_{\text{экв}}$. This is a force that creates the stress equal to sum from force and moment.

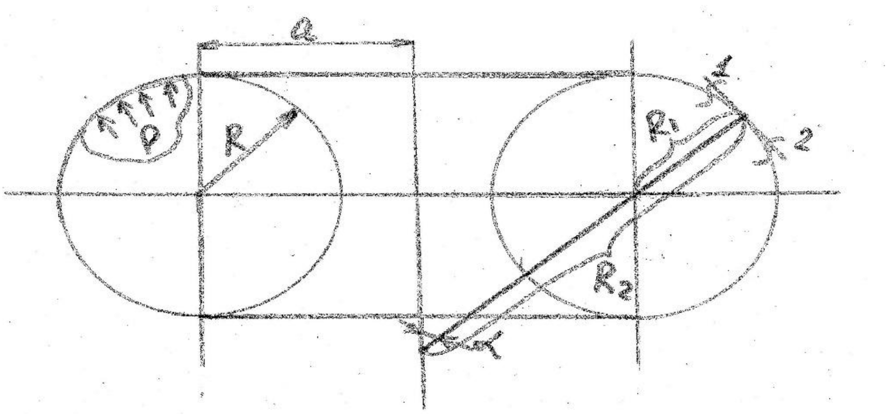
$$T_{\text{экв}} = T + \frac{2M}{R_{cylinder}}$$

$$\sigma_{\text{сж}} = \frac{T}{2\pi\, R_{cylinder} \delta_{cylinder}}$$

We are able to define the cylinder thickness from this strength analysis. However, compression condition calls the primary loss of geometry. Buckling phenomenon is happening that’s why strength analysis is insufficient and it’s necessary to perform the buckling phenomenon.

Object 5.

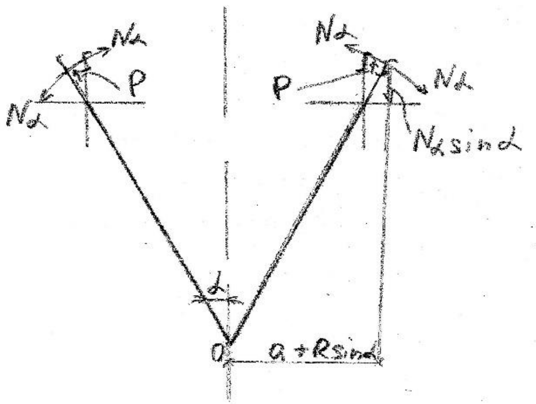
Torus shell calculation loaded by its own pressure



Torus shell is a shell defined by closed path rotation (sphere, ellipse etc.)

$$R_1 = R$$
$$R_2 = \frac{a}{\sin\alpha} + R$$

Considering the torus shell, it should be used the method of sections for finding of the internal forces.



Let's use static equations for line loads finding.

All forces sum to axial of symmetry is equal to zero:

$$\sum x: -2\pi(a + R\sin\alpha)N_2 \sin\alpha + Q = 0$$
$$Q_p = \int_0^{\alpha} 2\pi(a + R\sin\alpha)p\cos\alpha\, Rd\alpha$$

Having calculated the integral, we are getting:

$$Q_p = \pi p R \sin(2a + R\sin\alpha)$$

As a result, we are getting the following formula:

$$N_\alpha = \frac{PR}{2}\left(\frac{2a + R\sin\alpha}{a + R\sin a}\right)$$

Laplace's equation:

$$\frac{N_\alpha}{R_1} + \frac{N_\beta}{R_2} = P$$

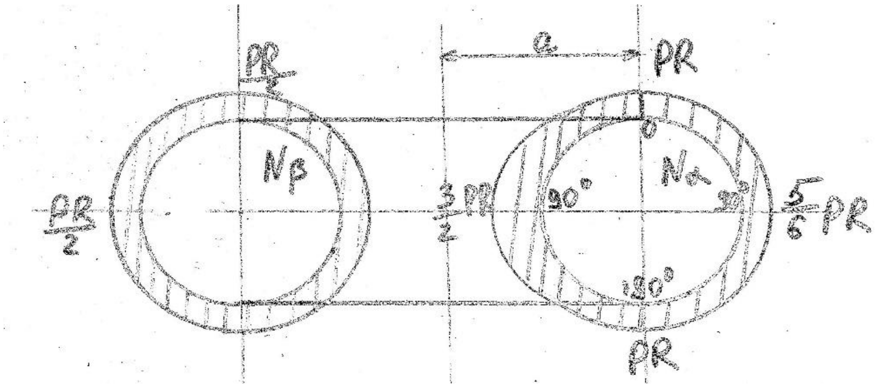
$$N_\beta = PR_2 = \frac{N_\alpha R_2}{R_1} = P\frac{a}{\sin\alpha} + R - \frac{PR}{2}\left(\frac{a + R\sin\alpha}{\sin\alpha}\right)\frac{1}{R}\left(\frac{2a + R\sin\alpha}{a + R\sin\alpha}\right)$$

$$= P\frac{1}{\sin\alpha}\left(a + R\sin\alpha - 2a\frac{1}{2} - \frac{1}{2}R\sin\alpha\right) = \frac{PR}{2}$$

$$N_\beta = \frac{PR}{2}$$

This result shows us that there is a non-homogeneous stress arising in the torus shell and there are also various meridian line loads arising in different points of torus.

Let's consider the example of distribution of internal line loads in torus



$$N_{\alpha=0} = PR$$

$$N_{\alpha=90^\circ} = \frac{PR}{2}\left(\frac{4a + R}{2a + R}\right) = \frac{5}{6}PR$$

$$N_{\alpha=180^\circ} = PR$$

$$N_{\alpha=-90^\circ} = \frac{PR}{2}\left(\frac{4a - R}{2a - R}\right) = \frac{3}{2}PR$$

Then, we are finding the stresses:

$$\sigma_{max} = \frac{N_{max}}{\delta}$$

$$\sigma_\alpha = \frac{PR}{2\delta}\left(\frac{2a + R\sin\alpha}{a + R\sin a}\right)$$

$$\sigma_\beta = \frac{PR}{\delta}$$

This characteristic property defines the manufacture singularity of torus tanks of two thicknesses.

This decision let us solve the check and design tasks, and also make lifting power check.

Check task:

$$\sigma_{\text{экв}I} = \sigma_{\text{экв}III} = \frac{PR}{2\delta}\left(\frac{2a-R}{a-R}\right) \leq [\sigma]$$

It's worth considering the safety factor impact f_p that is equal to 1,5.

$$\sigma_{\text{экв}III} = \frac{P^{э} f_p R}{2\delta}\left(\frac{2a-R}{a-R}\right) \leq [\sigma]$$

The structure is operative in case of:

$$n = \frac{[\sigma]}{\sigma_{\text{экв}}} \geq 1$$

Design task:

$$\delta \geq \frac{P^{э} f_p R}{2[\sigma]}\left(\frac{2a-R}{a-R}\right)$$

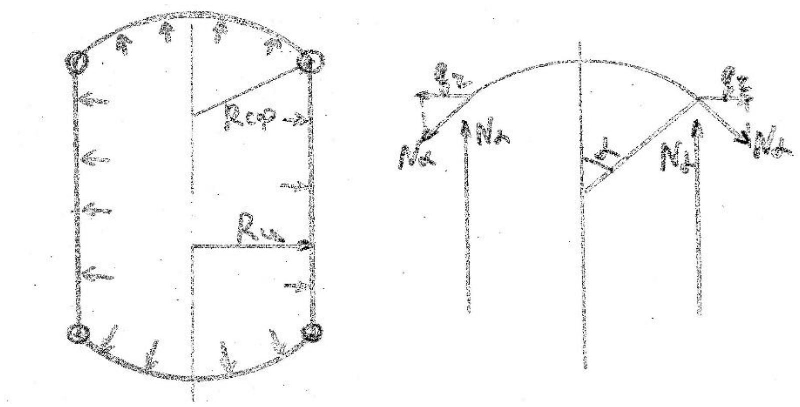
During design calculations performing, there are always some technological limits, namely $\delta \geq 1mm$.

Lifting power calculation consists in defining the service pressure where the construction will be operative**.** We are getting the following formula from the strength condition.

$$P^{\text{эксп}} = \frac{2\delta[\sigma]}{f_p R}\left(\frac{a-R}{2a-R}\right)$$

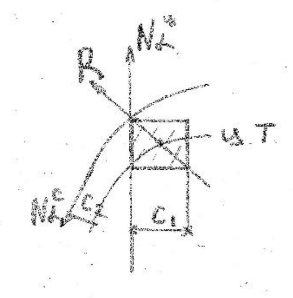
Spacer mating ring surface area definition

Adjunctive thrust forces arise in the tanks constructions in geometry changing area acting on a shell in weld junction.



$$q_r = N_\alpha \times Cos\alpha$$

Mating ring is fixed in thrust forces area that takes this thrust force. The choice of this mating ring has a set of features. Firstly, this ring has to possess such cross-section area so that the centroid of section will always be on a resultant of all operating forces of thrust forces area.



Secondly, cross-section area has to possess dimensions and center of gravity position where operating forces will not have to create the rotational moment.

$$N_\alpha^y \times c_1 + N_\alpha^c \times c_2 = 0$$
$$N_\alpha^y \times c_1 = N_\alpha^c \times c_2$$

These distinctive features let us make a choice of a spacer mating ring surface area from the mating ring operation condition with its cross-cut surface area.

$$q_r = N_\alpha^c \times Cos\alpha$$
$$q_r = \frac{P \times R_{\text{сф}}}{2} \times Cos\alpha$$
$$R_{\text{ц}} = R_{\text{сф}} \times Sin\alpha$$
$$R_{\text{сф}} = \frac{R_{\text{ц}}}{Sin\alpha}$$
$$q_r = \frac{P \times R_{\text{ц}}}{2 \times tg\alpha}$$

If the shell is in tension, so thrust forces q_r are in compression. We will determine the surface area spacer mating ring magnitude from asymmetrical ring strength condition. Let's consider the force magnitude from thrust forces arise constant stresses creating internal forces. Thrust forces magnitude in mating ring equal to stresses in mating ring multiplied by mating ring cross-cut surface area.

Internal forces magnitude is defined by section method from balance condition.

Resultant from thrust forces throughout the whole surface area equal to $2 \times q_r \times R$.

$$\sum x: -2 \times q_r \times R + 2\sigma_{\text{шп}} \times A_{\text{шп}}$$
$$\sigma_{\text{шп}} = \frac{q_r \times R}{A_{\text{шп}}}$$
$$A_{\text{шп}} = \frac{q_r \times R}{\sigma_{шп}} = \frac{P \times R^2}{2 \times \sigma \times tg\alpha}$$

The biggest stress that will stand the mating ring during the compression is a yield point, then:

$$A_{шп} = \frac{P \times R^2}{2 \times \sigma_т \times tg\alpha}$$

The mating ring surface area is found experimentally and has a view

$$A_{шп} = \frac{P \times R^2}{2 \times \sigma_т \times tg\alpha} - 0.788 \times \delta_ц \sqrt{R_ц \times \delta_ц} + \delta_{сф} \times \sqrt{R_{сф} \times \delta_{сф}}$$

$\delta_ц \sqrt{R_ц \times \delta_ц} + \delta_{сф} \times \sqrt{R_{сф} \times \delta_{сф}}$ - mating ring reduction of area due to combined action of cylindrical shell and tank-doomed bulkhead.

Stability checking of compressing ring is carried out as the mating ring works in compression. It is losing its stability with critical values of thrust forces.

$$q_r^{крит} = \frac{3 \times E \times I_{шп}}{R^3}$$

$$I_{min} \geq \frac{q_r^{крит} \times R^3}{3 \times E} = \frac{P \times R^4}{2 \times tg\alpha \times 3 \times E}$$

These results define the mating ring design algorithm:

1. Internal line forces are defined for predetermined constructions in shells jointing zone. Agreeably: these forces are set up pictorially in shells attachment point and find the equivalent force.
2. Mating ring surface area is analyzed according to conformities. This mating ring surface area is set up in resultant direction, besides, gravity center is situated on equivalent force action line, at the distance, where active line forces wouldn’t create the moment.

$$N_\alpha^y \times c_1 + N_\alpha^{сф} \times c_2 = 0$$

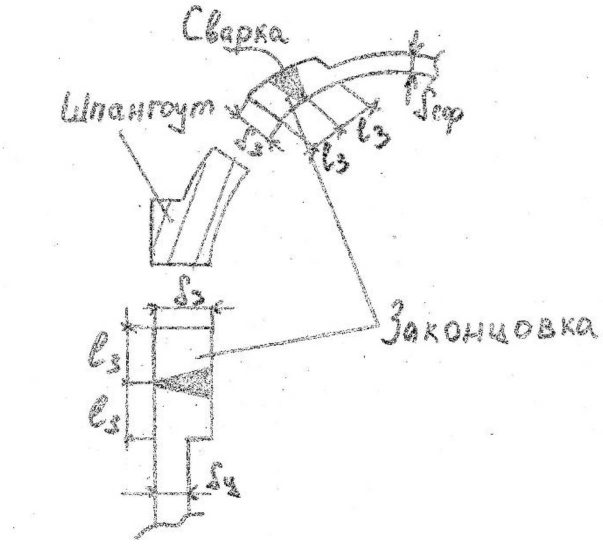
Stability testing is performed for the formality of mating ring cross-sectional area.

$$I_{min} \geq \frac{q_r^{крит} \times R^3}{36}$$

This algorithm allows us to implement the receiving of various forms of mating ring cross-section area.

The decision of aimed structure of mating ring cross-section area can be received only numerically.

Mating ring is attached with shells due to help of welding. That’s why shell edge line is performed in the tabbing zone.



$$\delta_{з} = 1.2 \times \delta_{св}$$
$$l_{з} = (5-7) \times \delta_{sphere}$$
$$\delta_{з} = 1.2 \times \delta_{cylinder}$$
$$l_{з} = (5-7) \times \delta_{cylinder}$$

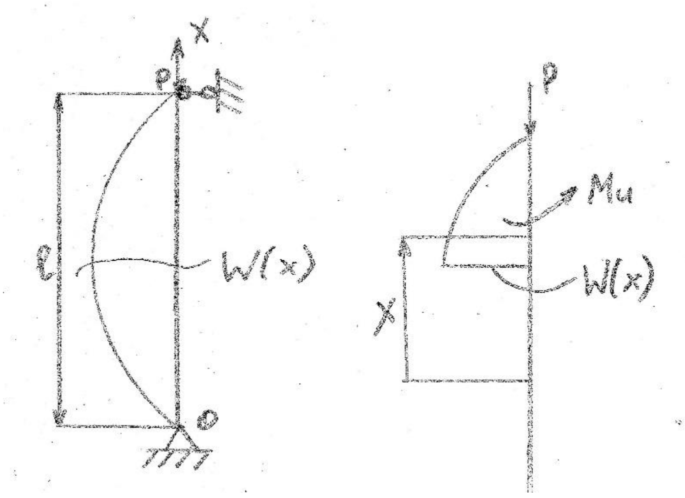
Aircraft structure elements buckling

The loss of elastic shakedown primary form of structure elements is happening under the compression force operation. Booms and stiffeners are structure elements that are fashioned in a beam model.

Let’s consider the bar buckling. Following buckling criterions are applied during the considering of load-bearing structure elements that are in compression. Buckling failure under the action of compression force arises in case when this force value exceeds critical forces. **Critical force** is some load that defines border between the stable and non-stable equilibrium state. The second criterion is based on comparison of operational stresses with the critical one. If the first stress is bigger than the second one, then buckling failure is happening.

Stress and critical force values determination

Let’s take a look at hinged bar. Length l and geometrical characteristics like: surface area A, second moment of area I and radios of inertia i are all known.



Let's define the value $F_{кр}$ and buckling failure for considering structure.

$$Mu = (EI)_{min}\frac{d^2w}{dx^2}$$

where EI – bending stiffness.

Balanced condition of defected axis part is provided by balance equation.

$$Mu - Pw(x) = 0$$
$$(EI)_{min}\frac{d^2w}{dx^2} + Pw(x) = 0$$
$$\frac{d^2w}{dx^2} + \frac{P}{(EI)_{min}}w(x) = 0$$
$$k^2 = \frac{P}{(EI)_{min}}$$

$$\frac{d^2w}{dx^2} + k^2(x) = 0$$

This differential equation describes the defected axis of balanced condition.
In this equation:

$$w(x) = A\cos kx + B\sin kx$$

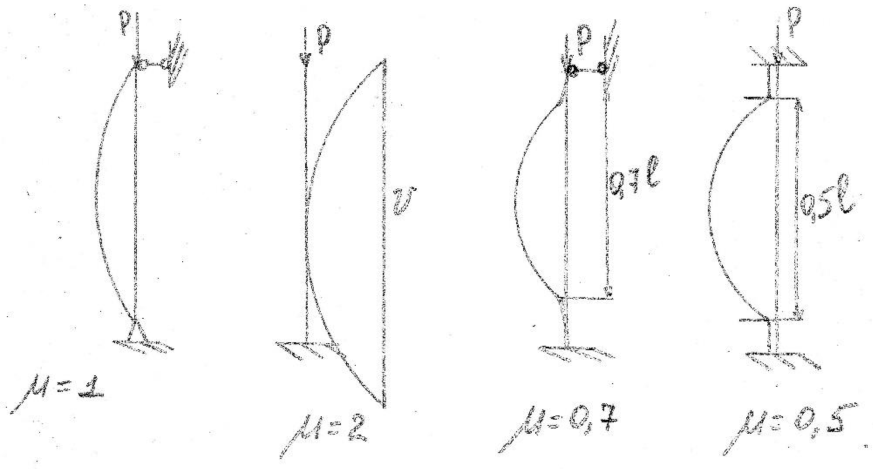
Now, we are using this formula for defining of the integration constant:

$$w(0) = A + 0 = 0$$
$$A = 0$$
$$w(x) = B\sin kx$$
$$k^2 = \frac{P}{(EI)_{min}}$$
$$k = \frac{n\pi}{l}$$
$$k^2 = \frac{P}{(EI)_{min}} = \frac{n^2\pi^2}{l^2}$$

$$P = \frac{n^2\pi^2(EI)_{min}}{l^2}$$

$$Px = \frac{n^2(EI)_{min}}{l^2}$$

Thus, we are getting **the Euler formula**.

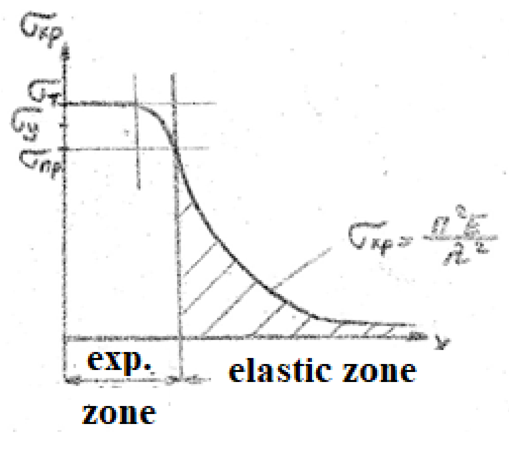


Generally, the formula can be defined:

$$P_{кр} = \frac{n^2(EI)_{min}}{\mu l}$$

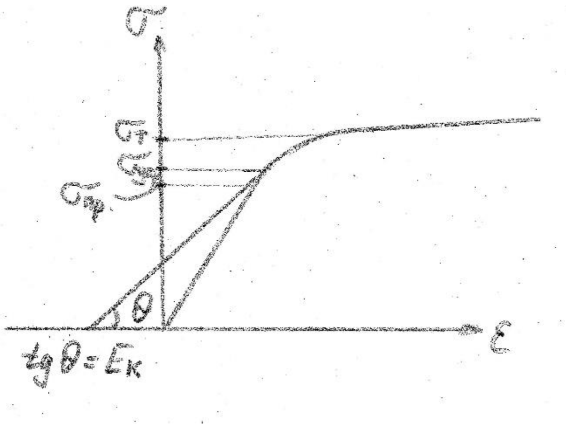
$$P_{кр} = \frac{n^2E}{\lambda^2}$$

where λ – slenderness.



Aluminum alloy is a main material in aircraft design practice. If there is a material diagram, so the value will be:

$$\sigma_{кр} = \frac{n^2E}{\lambda^2}$$



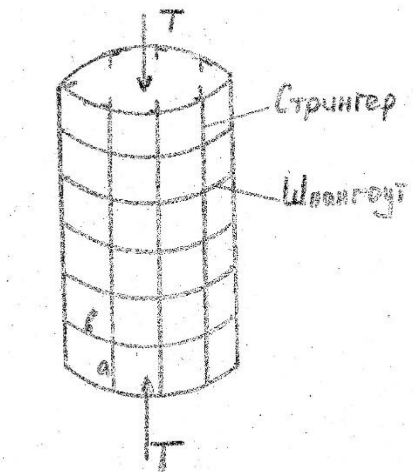
If there is no diagram, then the critical stresses are defined according to **Tetmayer formula**:

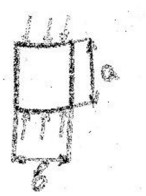
$$\sigma_{\text{кр}} = \sigma_{\text{т}}\left(1 - \left(1 - \frac{\sigma_{\text{п}}}{\sigma_{\text{т}}}\right)\sqrt{\frac{\sigma_{\text{п}}}{\sigma_{\text{т}}^{*}}}\right)$$

In case if $\sigma_{\text{кр}} > \sigma_{\text{п}}$, stress value is defined according to Tetmayer formula.
In case if $\sigma_{\text{кр}} < \sigma_{\text{п}}$, Euler formula is applied.

Plates buckling

Structure load-bearing elements are applied during the aircraft dry compartments consideration which called booms and mating rings. They make the typical form. Then, we are getting plate loaded by compression forces. Compression forces operating on a compartment call compression and elements possible buckling failure. Now, we are getting the plate a×b compressed in both sides by compressing line force P.

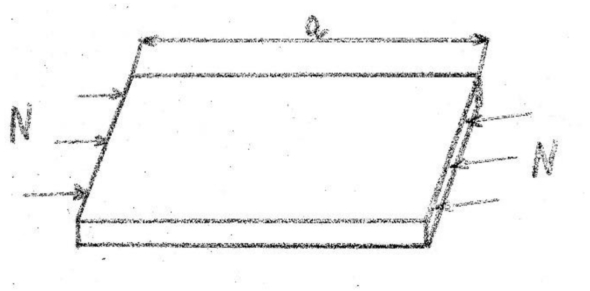




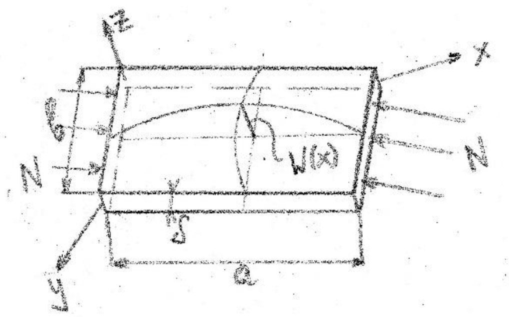
Primary form changing is happening under the operation of this load. Problem solving about plate buckling failure in a view of problem about the plate bending under some line load q action. Angles α_1 and α_2 are small.

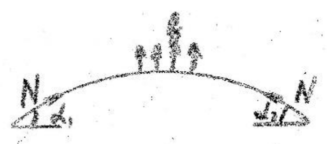
$$\sin \alpha_1 \cong \tan \alpha_1 = \frac{dw}{dx}$$
$$\alpha_2 = \alpha_1 + \frac{d\alpha_1}{dx} dx$$
$$\alpha_2 = \frac{dw}{dx} + \frac{d^2w}{dx^2} dx$$

Now, we are defining the critical compression force value N for the plate, where plate buckling failure is happening.



Compression line loads are equivalent to line force resultant q.





$$qdxdy = (N \sin \alpha_1 - N \sin \alpha_2)dy$$

We are making the following from this formula:

$$q = N \frac{d^2w}{dx^2}$$

This problem solving is leading to problem solving about rectangular plate bending under the action of pressure P. For that matter, we are treating to the known **Sophie – Germain – Lagrangian formula**.

$$\frac{d^4w}{dx^4} + \frac{2d^4w}{dx^2 dy^2} + \frac{d^4w}{dy^4} = \frac{q}{D}$$

where $D = \frac{E\delta^3}{12(1-\mu^2)}$

$$D\left(\frac{d^4w}{dx^4} + \frac{2d^4w}{dx^2 dy^2} + \frac{d^4w}{dy^4}\right) + N \frac{d^2w}{dx^2} = 0$$

$$q = -N \frac{d^2w}{dx^3}$$

Differential equation solving can be made with force of trigonometric sequences expanding:

$$w(x,y) = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y$$

This solving has the particular meaning for specified boundary conditions. If we insert this trigonometric solution into the changed Sophie – Germain – Lagrangian formula, we will get:

$$\frac{d^2w}{dx^2} = -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \frac{n^2\pi^2}{a^4} \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y$$

$$\frac{d^4w}{dx^4} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \frac{n^4\pi^4}{a^4} \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y$$

$$\frac{d^4w}{dy^4} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \frac{m^4\pi^4}{a^4} \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y$$

$$\frac{d^4w}{dx^2dy^2} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \frac{n^2\pi^2}{a^2}\frac{m^2\pi^2}{a^2} \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \frac{n^2\pi^2}{a^2}\frac{m^2\pi^2}{a^2} \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \left(D\left(\frac{n^4\pi^4}{a^4} + 2\frac{n^2\pi^2}{a^2}\frac{m^2\pi^2}{b^2} + \frac{m^4\pi^4}{b^4}\right) - N\frac{n^2\pi^2}{a^2}\right) \sin\frac{n\pi}{a} x \sin\frac{n\pi}{b} y = 0$$

Using the orthogonality property, we will modify trigonometric functions of this equation

$$D\pi^4\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^2 - N\frac{n^2\pi^2}{a^2} = 0$$

We can define the force from this equation that implements the plate bend and buckling failure:

$$N = D\frac{n^2\pi^2}{a^2}\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^2$$

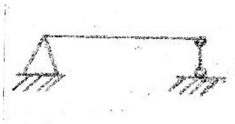
This force is a critical one where buckling failure is happening.

$$N = K_\sigma \frac{D\pi^2}{b^2}$$

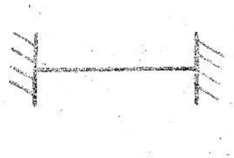
Where K_σ – is a plate fix condition coefficient.

Fix conditions define K_σ:

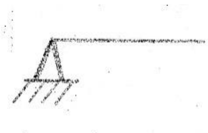
- It is equal to 4 for hinge bearing;



- It is equal to 7 during cantilever restraint;



- It is equal to 0,425 for free flap hinge;



- It is equal to 1,25 for free cantilever restraint.

The received formula is used for critical stress definition:

$$\sigma_{\text{кр}} = \frac{N_{\text{кр}}}{\delta}$$

$$\sigma_{\text{кр}} = 0{,}9K_{\sigma}\left(\frac{\delta}{b}\right)^2$$

For pin-edge ones:

$$\sigma_{\text{кр}} = 3{,}6K_{\sigma}E\left(\frac{\delta}{b}\right)^2$$

The received critical stress defines the buckling failure that arises in plate. This formula let us determine the applying zones of this formula or rather in elastic limit.
It means that every structure can possess such split $\frac{\delta}{b}$ that leads to structure operation outside the elastic rigidity:

$$\chi = \frac{\delta}{b}$$

$$\sigma_{\text{кр}} = \sigma_{\text{пу}} = 3{,}6K_{\sigma}E\chi^2$$

If there is no deformation diagram, then it is used:

$$A = 1 + \frac{1}{2}\frac{\sigma_{\text{п}}\sigma_{\text{т}}}{\sigma_{\text{п}}}\left(1 - \frac{\sigma_{\text{п}}}{\sigma_{\text{т}}}\right)^2$$

So, it’s necessary:

1. Check the critical stress value:

- if it is smaller than elastic limit:

$$\sigma_{кр} = 0{,}9K_\sigma \left(\frac{\delta}{b}\right)^2$$

- if the stress is higher:

$$\sigma_{кр} = \frac{\sigma_т}{A + \sqrt{A^2}}$$

2. The conformities receiving allow us to define the secondary buckling of aircraft buildup load-bearing elements. In this case K_σ is taken equal to 0,425 since it is always the free pin-edge fixing with force of splined fixing.

Aircraft booms and longerons check and design tasks

They are being solved based on two criterions:

1. According to acting compressing force and loading.
2. According to stresses.

Booms, longerons are fashioned due to trussed system that is why formulas for defining bar buckling failure are used for problems solving:

1. $P_{crytical} = \frac{\pi^2 (EI)_{min}}{(\mu l)^2}$
2. $\sigma_{crytical} = \frac{\pi^2 E}{\lambda^2}$

These formulas operate till elastic limit. Critical stresses are determined according to Tetmayer formula or to approximation method in case if they are situated from yield stress till elastic limit.

The following coefficient is put in for this:

$$\varphi = \frac{[\sigma_у]}{[\sigma]} = \frac{\frac{\sigma_{crytical}}{n_у}}{\frac{\sigma_т}{n}} = \frac{\sigma_{кр} n}{\sigma_т n_у} = \frac{\pi^2 E n}{\sigma_т n_у \lambda^2}$$

$$[\sigma_у] = \varphi[\sigma]$$

There is a slenderness dependency table from this coefficient for various materials.

Check task solution algorithm:

1. We are defining the surface area A, minimum inertial moment I_{min} , inertia radius i according to compression bar known geometry.
2. We are defining the effective length $l_{пр}$ and slenderness ration λ.

3. We are carrying out he critical force calculation $P_{кр}$.
4. Then, we are defining φ according to the table.

Design task algorithm:

1. Let's be designated by a coefficient φ, which is situated within limits of $0 \leq \varphi \leq 10$.
2. Now, we are defining the surface area from stability condition

$$A \geq \frac{P}{\varphi[\sigma]}$$

3. According to A, we are determining the diameter d and find i_{min}:
4.

$$A = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}}$$

$$i_{min} = \frac{d}{4}$$

5. Then, we defining the slenderness ratio λ according to known bar grip condition and inertia radius:

$$\lambda = \frac{\mu l}{i_{min}}$$

6. Now, let's use the linear interpolation according to the table.
7. Then, we are finding the thickness δ:

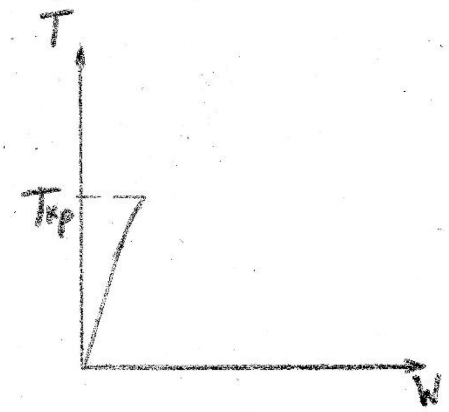
$$\delta = \frac{\varphi^{i-1} - \varphi^1}{\varphi^1} \times 100\%$$

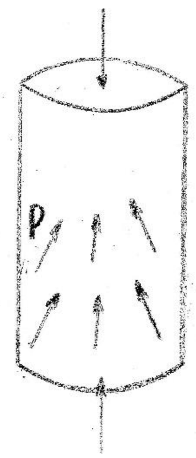
8. The actions are carried out iterative till the receiving of satisfactory performance $\delta \geq 1\%$. In case if the condition is not performed, we are getting back to section 2. The necessary result is usually made from 3-4 iterations.

Shells stability

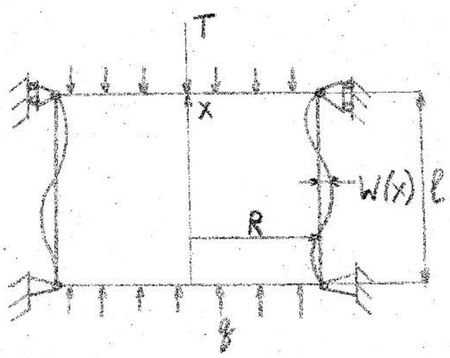
The loss of elastic equilibrium primary form compressing loads conditions is happened also in shells. In most times, such loads are compressive ones and external divided pressure.

From experiment, shells buckling failure are happening according to another conformities than in plates and bars. It appears by the fact that the dependence between loading called the buckling failure is the following: deformation dependence from loading during the loading till critical loads is linear; the deformation is increasing and the loading is decreasing during the buckling failure. The deformation is increasing it the plates and bars during the load increment.





Let’s consider the operation of hinged-supported cylinder for critical force finding where the buckling failure is happening.



Let’s consider that the symmetric buckling failure is happening.

Line inter radial forces are arises in these loading conditions $N_{\alpha} = q$.

Ring strength: $N_{\beta} = 0$

$$\sigma_{\alpha} = \frac{N_{\alpha}}{\delta} = \frac{q}{\delta}$$

Compressional cylinder strain stress state definition is carried out as a result of solidary consideration of static equations, geometric formulas of strain compatibility and physical formulas (elastic material correlation according to the Hook’s Law).

These equations blended decision let us make the problem to a single formula towards middle surface of a shell deflection:

$$D\frac{d^4w}{dx^4} + E\frac{\delta}{R^2} \times w + q\frac{d^2w}{dx^2} = 0$$

$$D = \frac{E\delta^3}{12 \times (1 - \mu^2)}$$

In case of hinged edge, fixing boundary conditions will be:

$$x = 0 \rightarrow w = 0$$

$$M = D \times \frac{d^2w}{dx^2} = 0$$

$$x = l \to w|_{x=l} = \frac{d^2w}{dx^2}|_{x=l} = 0$$

It is comfortable to seek the problem solving for these grip conditions in a view of trigonometric sequence:

$$w(x) = \sum_{m=1}^{\infty} A_m \sin\frac{m \times \pi}{l} \times x$$

Substitute in $D\frac{d^4w}{dx^4} + E\frac{\delta}{R^2} \times w + q\frac{d^2w}{dx^2} = 0$ and get:

$$\frac{d^2w}{dx^2} = -\sum_{m=1}^{\infty} A_m \sin\frac{m \times \pi}{l} \times x$$

$$\frac{d^4w}{dx^4} = \sum_{m=1}^{\infty} A_n\, sin\left(\frac{m \times \pi}{l}\right) \times A_m\, sin\frac{m \times \pi}{l} \times x$$

$$\sum_{m=1}^{\infty} A_n \times D \times \left(\frac{m \times \pi}{l}\right)^4 \times sin\left(\frac{m \times \pi}{l}\right) \times x + \sum_{m=1}^{\infty} A_m \times E \times \frac{\delta}{R^2} \times sin\left(\frac{m \times \pi}{l}\right) -$$

$$-\sum_{m=1}^{\infty} A_m \times \left(\frac{m \times \pi}{l}\right)^2 \times q_m \times sin\frac{m \times \pi}{l} \times x = 0$$

$$\sum_{m=1}^{\infty} A_m \times \left[D \times \left(\frac{m \times \pi}{l}\right)^4 + E \times \frac{\delta}{R^2} - \left(\frac{m \times \pi}{l}\right)^2 \times q_m\right] \times sin\frac{m \times \pi}{l} \times x = 0$$

$$A_m \neq 0;\ sin\frac{m \times \pi}{l} \times x \neq 0$$

$$D \times \left(\frac{m \times \pi}{l}\right)^4 + E \times \frac{\delta}{R^2} - \left(\frac{m \times \pi}{l}\right)^2 \times q_m = 0$$

$$q_m = D \times \left(\frac{m \times \pi}{l}\right)^4 + E \times \frac{\delta}{R^2} \times \frac{1}{\left(\frac{m \times \pi}{l}\right)^2}$$

Buckling failure is happening under the operation of line load q_m. It depends on parametric variable $\frac{m\times\pi}{l}$. Let's determine the smallest value:

Пусть $\left(\frac{m\times\pi}{l}\right)^2 = \lambda \to q_m = D \times \lambda + \frac{E\times\delta}{R^2\times\lambda}$

$$\frac{dq_m}{d\lambda} = 0 \to q_m = min \to \frac{dq_m}{d\lambda} = D - \frac{E \times \delta}{R^2 \times \lambda}$$

$$\lambda^2 = \left(\frac{m \times \pi}{l}\right)^4 = \frac{E \times \delta}{R^2 \times D}$$

$$\lambda = \left(\frac{m \times \pi}{l}\right)^2 = \sqrt{\frac{E \times \delta}{D}} \times \frac{1}{R} \to q_m = min$$

Now, let's substitute the found value $\left(\frac{m\times\pi}{l}\right)^2$ into the formula:

$$q_m = \sqrt{\frac{E \times \delta}{D}} \times \frac{1}{R} + E \times \frac{\delta}{R^2} \times \frac{R \times \sqrt{D}}{\sqrt{E \times \delta}}$$
$$q_m = \sqrt{\frac{E \times \delta \times D}{R^2}} + \sqrt{\frac{E \times \delta \times D}{R^2}} = \sqrt{\frac{4 \times E \times \delta \times D}{R^2}}$$
$$q_m = \frac{2}{R} \times \sqrt{E \times \delta \times D}$$

Stress, where the buckling failure will be presented:

$$\sigma_{crytical} = \frac{q_m}{\delta} = \frac{2}{R \times \delta} \times \sqrt{E \times \delta \times D} = \sqrt{\frac{4 \times E \times \delta \times D}{R^2}} = \sqrt{\frac{4 \times E \times E \times \delta^3}{R^2 \times \delta \times 12 \times (1 - \mu^2)}}$$
$$\sigma_{crytical} = \frac{E \times \delta}{R} \times \sqrt{\frac{1}{3 \times (1 - \mu^2)}} = \{\mu = 0.3\} = 0.605 \times E \times \frac{\delta}{R}$$
$$\sigma_{crytical} = 0.605 \times E \times \frac{\delta}{R}$$

This formula was received by S.P. Tymoshenko, it can't be confirmed experimentally. This thing is connected with the shell geometry idealization.

$$\sigma_{crytical} = K_R \times E \times \frac{\delta}{R}$$
$$K_R = 0.1 - 0.2$$

This result was used for compressing cylinder critical stresses analysis under the act of axial force. Imperfection value K_R is defined on the basis of experiments in engineering calculations.

It's used in different sources:

$K_R = \frac{1}{\pi} \times \sqrt[8]{\left(\frac{100 \times \delta}{R}\right)^3}$ - Lizin-Pyatkin

$K_R = 0.605 \times K_c$ – Mosakovsky

Unstiffened shells imperfection value $(0.1 - 0.2)K_R$, for stiffened shells (booms) $(0.4 - 0.5)K_R$, for honeycombs $(0.7 - 0.8)K_R$.

This conformity let us perform the check and design ones and define the lifting power.

Check calculation:

The calculation determines K_R and $\sigma_{crytical} = K_R \times E \times \frac{\delta}{R}$ according to it for given geometry,

$$T = 2 \times \pi \times R \times \delta \times \sigma_{кр} = 2 \times \pi \times R \times \delta \times K_R \times E \times \frac{\delta}{R}$$
$$T = 2 \times \pi \times K_R \times E \times \delta^2$$
$$T^P \leq T_{crytical}$$

$\eta = \frac{T_{кр}}{T^P} \geq 1$ – the structure is resistant.

Design calculation:

Start value $K_{R0} = 0.18$ is designated during its performing. Then, we are determining δ from stability condition in handbook calculation:

$$\delta^{(1)} = \sqrt{\frac{T^P}{2 \times \pi \times K_R \times E}}$$

$$K_R^{(1)} = \frac{1}{\pi} \times \sqrt[8]{\left(\frac{100 \times \delta}{R}\right)^3}$$

$$\frac{K_R^i - K_R^{i-1}}{K_R^{i-1}} \times 100\% < 1\%$$

If the condition is not carried out, so the iterations are being performed until the condition is made.

This iteration procedure converges through 5-6 iterations.

Lifting power determining:

Compressing cylinder lifting power value defines the buckling failure critical load:

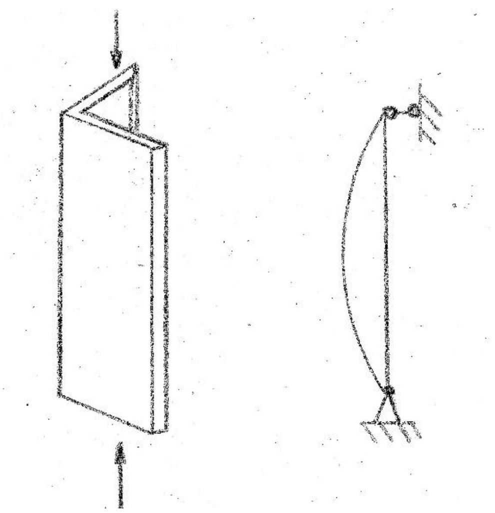
$$T_{crytical} = 2 \times \pi \times K_R \times E \times \delta^2$$

The received solving allows us to give some recommendations at the material choice and structure geometry from the stability condition.

General and local buckling. Cylindrical shell buckling analysis.

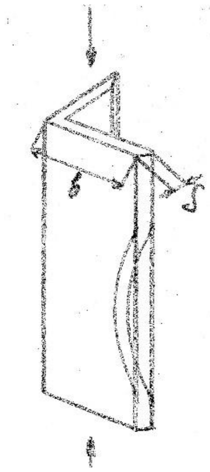
General buckling is elastic element all form loss (compression bar).

Local buckling is a changing local form with form loss in compressing structure conditions.



$$P_{crytical} = \frac{\pi^2 \times E \times I_{min}}{(\mu \times l)^2}$$

$$\sigma^{o}_{crytical} = \frac{\pi^2 \times E}{\lambda} = \frac{\pi^2 \times E}{\left(\frac{\mu \times l}{i_{min}}\right)^2}$$



$$\sigma^{\text{м}}_{crytical} = K_{\sigma} \times E \times \left(\frac{\delta}{b}\right)^2$$

$K_{\sigma} = 0.425$ - anchorage coefficient.

Profiles standard set used in rocket engineering provides about 15 per cent excess of local buckling under the stresses of general buckling, but we should always perform the local and general buckling calculation.

$$\eta^{\text{о}}_{\text{у}} = \frac{\sigma_{crytical}}{\sigma_{\text{р}}} \geq 1$$

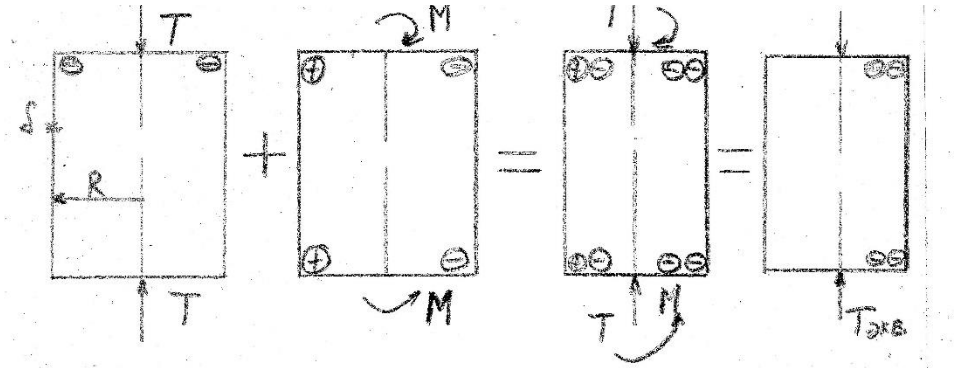
$$\eta^{\text{м}}_{\text{р}} = \frac{\sigma^{\text{м}}_{crytical}}{\sigma_{\text{р}}} \geq 1$$

Cylindrical shell stability in bending moment and axial force combined action conditions

Cylindrical shell as a tank element is situated in loads combined action conditions in peak dynamic pressing zone during the aircraft flight.

Cylindrical shell stability analysis is performed according to equivalent load value in this simulation case.

Equivalent load (force) is a compressing force that calls such stresses which equal to stresses sum force from axial force and bending moment in rocket airframe slicing.



$$\sigma' = \frac{T}{2\times\pi\times R\times\delta} \qquad \sigma'' = \frac{M}{w}$$

$$\sigma_{equivalent} = \frac{T_{экв}}{2\times\pi\times R\times\delta} = \sigma' + \sigma'' = \frac{T}{2\times\pi\times R\times\delta} \pm \frac{M}{\pi\times R^2\times\delta}$$

$$T_{equivalent} = T + \frac{2\times M}{R}$$

Aircraft airframe or cylindrical propellant tank buckling analysis in force and moment combined action conditions are carried out after the equivalent force.

Checking calculation, design calculation and lifting power calculation are performed analogically after equivalent load in this case.

$$\sigma_{crytical} = K_R \times E \times \frac{\delta}{R}$$

$$T_{equivalent} = 2\times\pi\times K_R\times\delta^2\times E$$

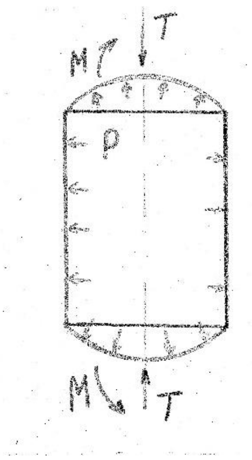
$$\delta = \sqrt{\frac{T_{экв}}{2\times\pi\times K_R\times E}} = \sqrt{\frac{T + \frac{2\times M}{R}}{2\times\pi\times K_R\times E}}$$

$$\delta = \sqrt{\frac{T^{э}\times f_T + \frac{2\times M^{э}\times f_M}{R}}{2\times\pi\times K_R\times E}}$$

$$f_T = f_M = 1.3$$

Buckling analysis is carried out after the highest equivalent loads during the tank design for the second and the third simulation cases.

Cylindrical shell axial compression considering the manifold pressure



Integral tanks can be located simultaneously under the pressure of axial compression force, bending moments and internal pressing.

Operational internal pressuring makes the structure load alleviation.

$$F_{разгр} = P_{min} \times \pi \times R^2$$
$$P_{min} = P_{н} - \Delta P$$
$$\Delta P = 0.1 Мпа$$

This of-loading force leads to compression load decreasing that acts on a tank and, also fluff the shell making it the dome-shaped oneб thereby decreasing the flaws and discrepancies of cylinder shell.

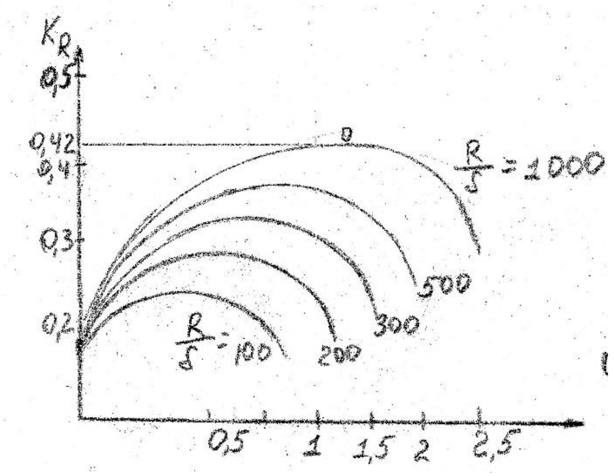
This scene is confirmed experimentally.

Cylindrical shell calculation is performed after the same formulas as the cylinder.

$$\delta = \sqrt{\frac{T^{э} \times f_T + \frac{2 \times M^{э} \times f_M}{R} - \pi \times R^2 \times P_{min}}{2 \times \pi \times K_R \times E}}$$
$$\sigma_{кр} = K_R \times E \times \frac{\delta}{R}$$
$$T_{equivalent} = 2 \times \pi \times K_R \times \delta^2 \times E$$

In these formulas K_R – imperfection value is defined based on analysis experiments pictorially or analytically according to the following formulas:

$$\bar{P} < 0.8 \rightarrow K_R = K_0 + 0.265 \times \sqrt{\bar{P}}$$
$$\bar{P} > 0.8 \rightarrow K_R = 0.42$$



These formulas are used for checking and design calculations performing. Stability coefficient is a performance criterion during checking calculations performing:

$$\eta_{у} = \frac{T_{кр}}{T_{р}} \geq 1$$

$$T^{Р} = T^{Р}_{экр} = T^{э} \times f_T + \frac{2 \times M^{э} \times f_M}{R} - \pi \times R^2 \times P_{min}$$

$$\eta_{у} = \frac{\sigma_{crytical}}{\sigma_{р}} \geq 1$$

Operative structure geometric parameters are defined by iteration method during design tasks performing after the scheme:

$$K_R^0 = 0.18$$

$$\delta_0 = \sqrt{\frac{T_{экв}}{2 \times \pi \times K_R \times E_t}}$$

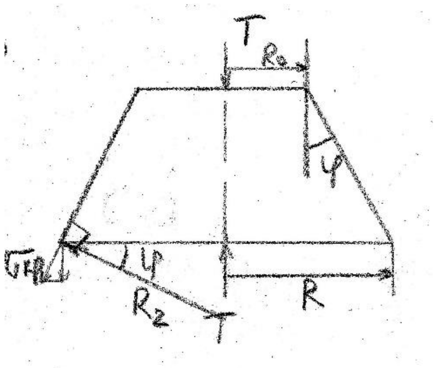
$$K_R{}^{(i)} = \frac{1}{\pi} \times \sqrt[8]{\left(\frac{100 \times \delta}{R}\right)^3}$$

if $\frac{K_R^i - K_R^{i-1}}{K_R^{i-1}} \times 100\% < 1\%$- the calculation is finished

if $\frac{K_R^i - K_R^{i-1}}{K_R^{i-1}} \times 100\% > 1\%$ - so, the pressure is re-counted

$P = \frac{P_{н}}{E} \times \left(\frac{R}{\delta}\right)^2$, and then we are making the whole iteration re-counting until the condition $\frac{K_R^i - K_R^{i-1}}{K_R^{i-1}} \times 100\% < 1\%$ is performed.

Conical shell buckling analysis affected by compressing



As the experiment shows, buckling failure in conical shell is happening as a result of shell cotton-shaped crippling in the maximum radius zone.

Buckling failure from compression force can be determined according to experiment. Out from the experiment, conical shells buckling failure beginning with cone half angle 10˚-60˚can be described well by formulas for cylindrical shells with $R_{ц} = R_2$.

$$\sigma_{crytical} = K_R \times E \times \frac{\delta}{R_2} = K_R \times E \times \frac{\delta}{R} \cos\varphi$$

$$K_R = \frac{1}{\pi} \times \sqrt[8]{\left(\frac{100 \times \delta}{R}\right)^3}$$

$$K_R = 0.605 \times K_{c}$$

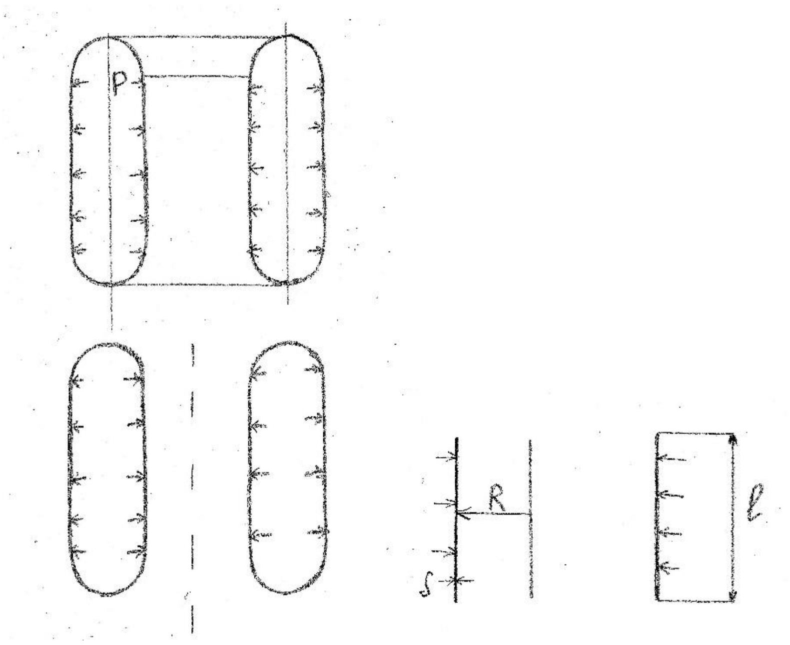
$$K_{c} = 1 - 0.9 \times \left(1 - exp\left(-\frac{1}{16}\sqrt{\frac{R}{\delta}}\right)\right)$$

$$T = \sigma_{crytical} \times cos\,\varphi \times 2 \times \pi \times K_R \times \delta = 2 \times \pi \times K_R \times \delta^2 \times (cos\,\varphi)^2$$
$$T_{кр} = 2 \times \pi \times K_R \times \delta^2 \times (cos\,\varphi)^2$$
$$\eta_{у} = \frac{T_{кр}}{T_{р}} \geq 1$$
$$\delta = \sqrt{\frac{T_{кр}}{2 \times \pi \times K_R \times (cos\,\varphi)^2}}$$

If the cone half angle is 0˚-10˚, then

$$R_R = \frac{R + R_0}{2} \times \frac{1}{cos\,\varphi}$$
$$\sigma_{crytical} = K_R \times E \times \frac{\delta}{R_R}$$

Cylindrical shell buckling affected by external differential pressure



Aircraft structure members can lose its stability under the influence of external differential pressure (fairing, cylinder).

Each structure member buckling failure is defined by geometrical parameters and material mechanical characteristics.

Let’s consider the cylinder buckling failure with radius R, length l, and thickness δ. Critical pressing value where the buckling failure for cylinder is happening is determined according to Popkovich formula:

$$P_{crytical} = K_c \times \frac{\pi \times \sqrt{6}}{9 \times (1 - \mu^2)} \times E \times \left(\frac{R}{l}\right) \times \left(\frac{\delta}{R}\right)^{\frac{5}{2}}$$
$$P_{crytical} = K_c \times E \times 0.92 \times \left(\frac{R}{l}\right) \times \left(\frac{\delta}{R}\right)^{\frac{5}{2}}$$

$K_c = 0.7 - 0.8$ - cylindrical shell figure of merit. It was received based on experiments and it specifies the critical pressing value according to Popkovich formula.

$$\sigma_{кр} = \frac{P_{кр} \times R}{\delta}$$

The critical stress corresponds to buckling failure critical pressing:

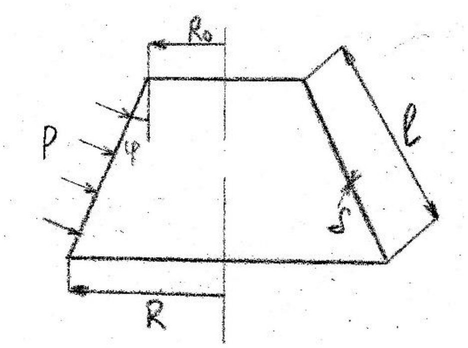
$$\sigma_{кр} = \frac{P_{кр} \times R}{\delta} \times K_c \times E \times 0.92 \times \left(\frac{\delta}{l}\right) \times \sqrt{\frac{\delta}{l}}$$

This formula is used for checking, design and lifting power calculations.

Stability criterions are condition:

$$\eta_у = \frac{P_{кр}}{P_р} \geq 1$$

Conical shell buckling analysis loaded by external differential pressure



Buckling failure critical pressure value is defined by Popkovich model and is specified based on experiment, cone half angle influence coefficient.

If the cone half angle is 10˚-25˚, then

$$P_{crytical} = K_c \times E \times 0.92 \times \left(\frac{R}{l}\right) \times \left(\frac{\delta}{R}\right)^{\frac{5}{2}} \times (\cos\varphi)^{\frac{3}{2}}$$

If the cone half angle is 25˚-70˚, then

$$P_{crytical} = K_c \times \rho \times E \times 0.92 \times \left(\frac{R}{l}\right) \times \left(\frac{\delta}{R}\right)^{\frac{5}{2}} \times (\cos\varphi)^{\frac{3}{2}}$$

$$\rho = 3.1 - 2.47 \times \frac{R_0}{R} \to 0 \leq \frac{R_0}{R} \leq 0.6$$

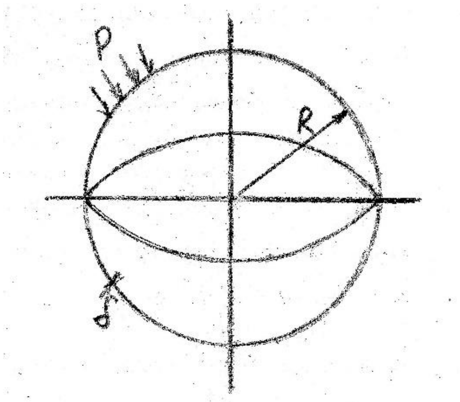
$$\rho = 2.66 - 1.74 \times \frac{R_0}{R} \to 0.65 \leq \frac{R_0}{R} \leq 1$$

If the cone half angle is 0˚-10˚, then

$$P_{crytical} = K_c \times E \times 0.92 \times \left(\frac{R^*}{R}\right) \times \left(\frac{\delta}{R^*}\right)^{\frac{5}{2}}$$

$$R^* = \frac{R_0 + R}{2}$$

Spherical shell buckling failure under the action of external differential pressure



As the experiment shows, spherical shell buckling failure under the action of external differential pressure is happening during the stresses when their value coincides with cylindrical shell critical stresses under the action of compression force.

$$\sigma_{crytical} = K_R \times E \times \frac{\delta}{R}$$
$$K_R = 0.605 \times K_c$$

$$K_c = 1 - 0.9 \times \left(1 - exp\left(-\frac{1}{16}\sqrt{\frac{R}{\delta}}\right)\right)$$

$$P_{crytical} = \frac{2 \times \sigma_{кр} \times \delta}{R} = 2 \times K_R \times E \times \delta^2$$

$$\sigma_{crytical} = \frac{P \times R}{2 \times \delta}$$

Buckling failure is determined:

$$\eta_y = \frac{P_{кр}}{P_р} \geq 1$$

These formulas are used for spherical segments buckling failure determining (provisional aft ends) under the action of external pressing.

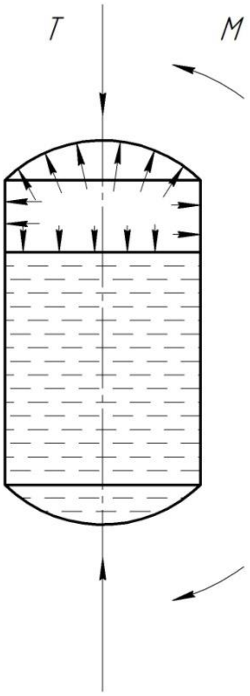
As the experiment shows, spherical segment buckling failure is 30% smaller than sphere one, that’s why:

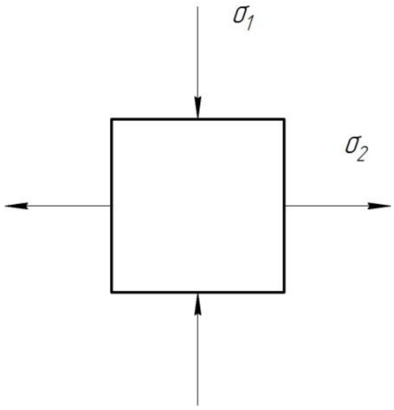
$$\sigma_{crytical} = K_3 \times K_R \times E \times \frac{\delta}{R}$$
$$K_3 = 0.7$$
$$K_R = 0.605 \times K_c$$
$$P_{crytical} = 2 \times K_3 \times K_R \times E \times \delta^2$$

Waffle – grid structure tank wall calculation

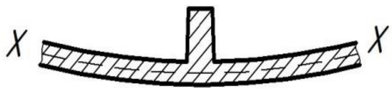
Analytical model

Let's consider the waffle – grid structure.
Analytical model: cylindrical shell is loaded by: T, M, $P_{н}$, $P_{г}$, t^{o}.



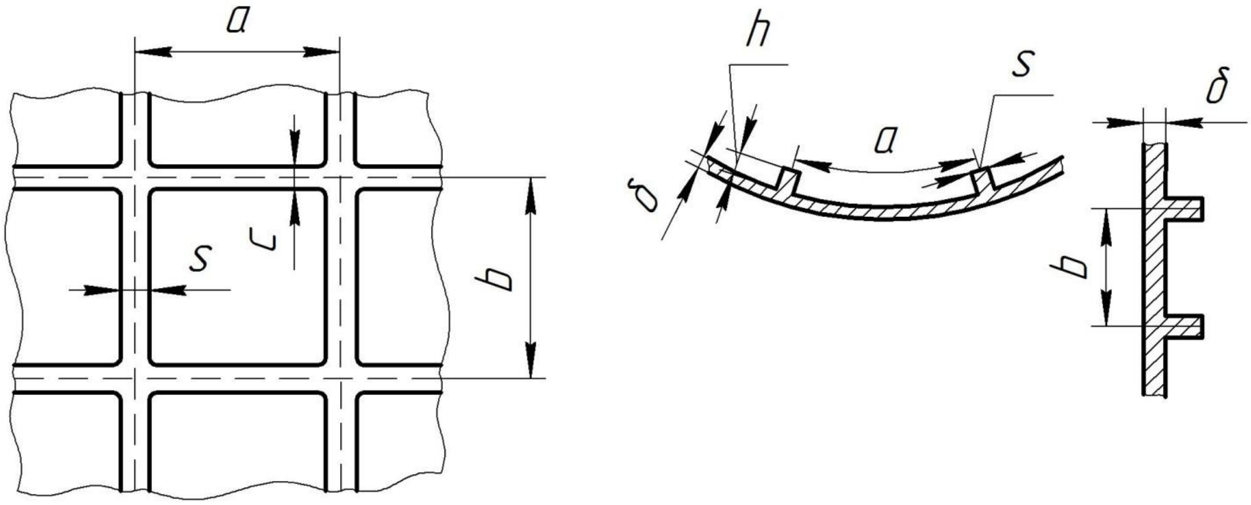


Unstiffened shells operate too ineffectively during the action of axial compression..
$\sigma_{кр}/\sigma_{т} = 0.1\text{-}0.2$
Longitudinal stringers enforcement increases the lifting power (збільшується shell cylindrical stiffness is increasing and the mass is distributed more rationally). Transversal ribs (mating rings) increases the stringer lifting power, in which case it increases the extensional crossing effective thickness that operates on integrity in cross direction. Besides, Waffle shells are less quick-response to initial imperfections.



Waffle shell lets decrease the compartment mass by two or three times. Square case is used the most often that is featured by four characteristics:

δ – fabric thickness;
h – fin length (or primary thickness);
S=C – fin width;
a=b – cage dimensions, fins distance.



Such shell can be considered as a structurally orthotropic one in the handbook calculation during strength analysis and general storage qualities (ribs conditional distribution).

This is a more exact approach of computational methods application, as an example – finite – element method that is in need of materials application and they are not very comfortable for design analysis.

Structurally – orthotropic shell is featured by characteristics:

$\delta_c = \delta + \frac{F_c}{a}$ - inverse intersection effective thickness that is taking up axial stresses;

$\delta_ш = \delta + \frac{F_ш}{b}$ - axial intersection effective thickness that is taking up hoop stresses;

F_c and $F_ш$ - booms and mating rings surface area accordingly;

D_c and $D_ш$ - elements cylindrical stiffness of inverse and axial intersection accordingly.

Operational compression stresses:

Axial compressional stresses: $\sigma_1 = -\frac{T^p_{екв} - P_{min}\pi R^2}{2\pi R\delta_c}$

Hoop stresses: $\sigma_2 = \frac{P^p R}{\delta_ш}$

Operational stresses and possible shell fracture modes

Buckling failure

General buckling is happening by handling marks creation in circular direction with some pallets catching along with stiffening ribs.
Axisymmetric and symmetric forms of failure storage qualities are possible. General storage qualities critical stresses can be determined in accordance with structurally orthotropic shell scheme:

$$\sigma^{з}_{кр} = k_c\sqrt{\frac{4E_tD_ш}{R^2\delta_c}}\ ,\ T_{кр} = 2\pi R\delta_c\sigma^{з}_{кр} = 4\pi k\sqrt{E_tD_шh_c}$$

k_c – durability coefficient. It's recommended to take k_c = 0,27 - 0,3 for waffle shells.

Critical stresses of durability failure axisymmetric and symmetric forms are equal for square cases.

Fabric local durability failure between the ribs in a separate case is below. Holding in scorn by fabric surface curvature in a separate case like an analytical model for an outer cover, we are able to accept plate set up strengthened along the edges. Plate edges strengthening are situated between the guiding fin and jamming according to test information.

$$\sigma^{м}_{кр.п.} = k_пE_t\left(\frac{\delta}{a}\right)^2,\ k_п = 6{,}7$$

It's worth noting that handling marks appearance between them don't lead to lifting power exhaustion of stiffened shell, but decreases by 15 – 20% $\sigma^{з}_{кр}$ with sufficient strict ribs. From this perspective, it's desirable to be provided with the condition

$$\sigma^{м}_{кр} \geq \sigma^{з}_{кр}\,.$$

Stringer local storage qualities failure is possible for separates types of waffle shell (with sizable ribs). Analytical model is a compressional plate where its first edge is free and another one is leaned (in material factor). Then: $\sigma^{м}_{кр.с} = k_пE_t\left(\frac{c}{h}\right)^2,\ k_п = 0{,}38$.

It's worth paying attention to the fact that the last three formulas operates only in elastic region. It is possible beyond the elasticity edges:

а) approximate empirical equations applying;
б) calculations according to contacting elasticity modulus carrying out:

$$E_t = E\tau\,,\ \tau = \frac{(\sigma_п - \sigma)\sigma}{(\sigma_т - \sigma_п)\sigma_п}\,.$$

σ - actual stresses, $\sigma_п$ - proportionality edge, $\sigma_т$ - yielding limit.

Strength loss has its place if axial stresses reach the parameter $\sigma_т$ and in circular direction - $\sigma_в$, so the strength condition looks like:

$$\sigma_1 \le \sigma_{т,t}, \ \sigma_{екв} \le \sigma_{в,t}.$$

According to the third failure theory, it is going to be like

$$\sigma_{екв} = \sigma_2 + |\sigma_1| \to \sigma_2 + |\sigma_1| \le \sigma_{в,t}.$$

Waffle tank wall design analysis. (Optimization task statement)

It's given: dimensional specifications R,L. Loads like T,M,$P_н$,$P_г$,t^o in suitable simulation cases material AMg6M or AMg6NN.
It's requires: to determine the waffle wall characteristics that provides the tank functional capability and minimum mass (square case, $\delta, \delta_{iсх}$, C=S, a=b).

I Tank non-destroying conditions:

Operational stresses:

$$\sigma_1 = \frac{T^p_{екв} - \pi R P_{min}}{2\pi R \delta_c}$$

$$\sigma_2 = \frac{P^p_{max} R}{\delta_c} = \frac{P^p_{max} R}{\delta_ш}$$

1) Storage qualities conditions

$$\sigma_1 \le \sigma^з_{кр}$$

$$\sigma_1 \le \sigma^м_{кр.п}$$

$$\sigma_1 \le \sigma^м_{кр.с}$$

2) Strength conditions:

$$\sigma_1 \le \sigma_{T,t}$$

$$\sigma_{екв} \le \sigma_2 + |\sigma_1| \le \sigma_{T,t}$$

II Object function is a shell mass. The characteristics that provide the M_{min} are being determined.

$$M = 2\pi R L \rho \delta_e$$

δ_e- conventional unstiffened shell equivalent thickness where its mass equals to waffle shell mass.

$$\delta_e = \delta + \frac{Sh}{a} + \frac{Ch}{b} - \frac{CSh}{ab} = \delta + \frac{Ch}{a} + \frac{(a-C)Ch}{a^2}$$
$$\delta_e \approx \delta + \frac{2C(\delta_{icx} - \delta)}{b} = \delta + \frac{2Ch}{a}$$

III It's necessary to consider the technological and structure pattern qualifications during the objective structure characteristics defining. These characteristics can be entitled to qualifications:

- original plate thickness $\delta_{icx} \le \delta_{icx.max}$
- plating thickness $\delta \ge \delta_{техн}$
- ribs thickness $C = S \ge C_{техн} = S_{техн}$

At this rate, optimization problem arises: we have to determine the waffle shell characteristics δ, δ_{icx}, S=C, a=b along with the given envelope, loads and materials that provide the minimum of a function $M = M(\delta, \delta_{icx}, C, h)$ during the non – destroying conditions performing and by considering the qualifications.

This is a complex multivariable optimization problem (became known as, nonlinear programming problemтак звана). Various optimization algorithms can be applied for its solving: random search method, gradient method and others. These are difficult calculating algorithms that are in need of personal computer using.

Along with that, there are some approximate approaches that allow us to determine the shell characteristics close to optimized one relatively simple. These approaches are applied during the preliminary design phases when it's essential to determine all the shell geometric characteristics, to specify the compartment mass and the whole aircraft depending on acting load. When this happens, various design alternates like unstiffened one, panelized – boom one and waffle one are being considered.

We have familiarized with one of such approximate approaches during the practice time by using book of V.T. Lyzin, V.A. Pyatkin which is called "Thin-slab structure design".

Panel immunity reinforced by axial framework

Plates that are used in aircraft structures with a view of critical load building up as a rule are ribbed with an axial framework like stringers and in some cases with frames.

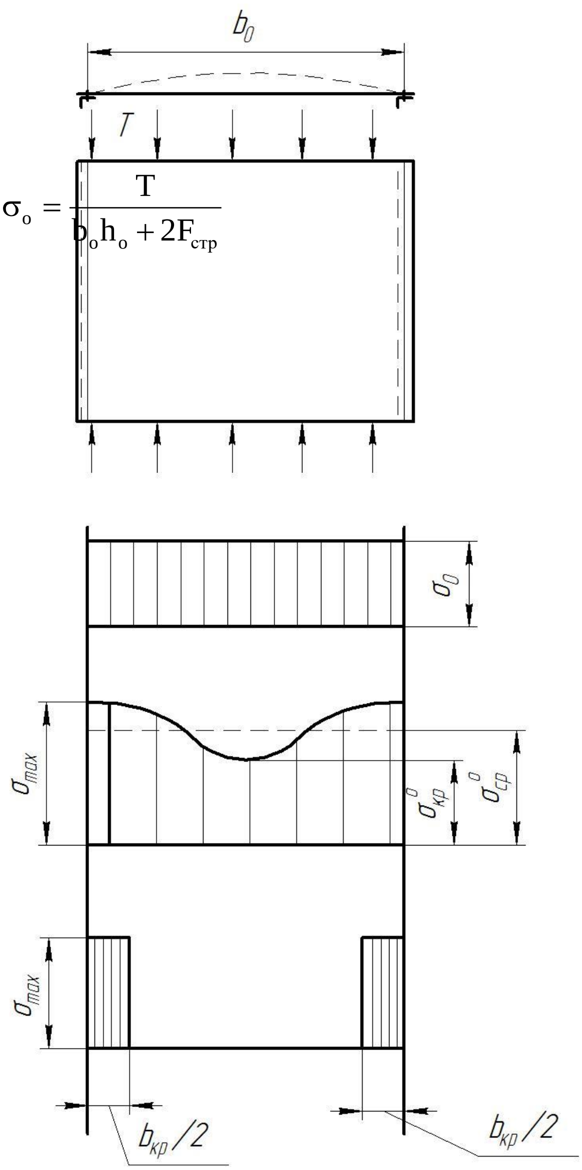
Such structures are usually called panels. Panels can be both planed and curved.

Immunity loss among the stringers for the whole structures set is acceptable. For example, oblate wing part envelope can receive the craters placed among the stringers.

Thin envelope immunity loss is also acceptable for the stringer structure dry compartment in most cases. These actions have to be considered during the structural analysis.

Let’s consider approximately the way of envelope loading evaluation that has lost its immunity into the panel lifting power.

Let’s consider the flat panel reinforced by two stringers. The panel is loaded by distributed load. «T» is a this load resulting force.



$$\sigma_o = \frac{T}{b_o h_o + 2F_{стр}}$$

As long as the load is not big enough, the stress divides up all along the panel width intimately and being determined according the equation:

$$T = \sigma_o \left(b_o h_o + 2F_{стр} \right)$$

With the increase of load T, the stress σ_o increases by reaching the value $\sigma^o_{кр}$ and the envelope losing its immunity.

$$T' = \sigma^o_{кр} \left(b_o h_o + 2F_{стр} \right)$$

T' is a load magnitude where the envelope will lose its immunity.

Under $T > T'$ the loads are taken up generally by stringers, but the envelope part that is adjacent to stringers keeps on operating.

Stress to envelope immunity loss divides up intimately and they have the minimum in the panel mid-point and reach its maximum values along the edges after the immunity loss.

If the envelope and stringer are made from the same material and connected by welding or densely spaced rivets, so the stresses on the envelope edges and in stringers are the same.

In order to determine the load accommodation by a plate after envelope immunity loss, it’s necessary to know the exact stress distribution law which can only be found as a result of complex problem solving about hypercritical plate behavior.

Different approximate approaches are applied for approximate estimate of envelope loading that has lost the immunity in the general lifting power.

The most statistically reliable one turns out to be the following:
Averaging out the undetermined distribution all along the plate width according to geometric mean rule, so considering that the whole plate operates with average stress:

$$\sigma^{o}_{ср} = \sqrt{\sigma^{o}_{кр}\sigma^{o}_{max}}$$

Then, the loads that are taken up by the panel can be written:

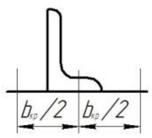
$$T = 2F_c\sigma^c + b_o h_o \sigma^{o}_{ср}$$

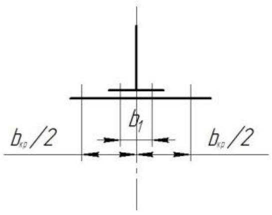
If the envelope and stringer material are identic, so it will be $\sigma^{o}_{т} = \sigma^{c}$.

Panel destroying is connected with the boundary stresses attainment in stringers. And the panel lifting power is consist of a plate and stringers lifting power:

$$T_{нес} = 2F_c\sigma^{c}_{гран} + b_o h_o \sigma^{o}_{ср}, \text{ where } \sigma^{o}_{ср} = \sqrt{\sigma^{o}_{кр}\sigma^{o}_{гран}}.$$

During practical analysis, we are able to take into account the envelope operation process approximately in another way and evaluate the panel lifting power:





Let's tale the fact that the envelope center portion which has lost the immunity don't operate at all, but in the part of the plate with width $b_{кр}$, the stresses effect $\sigma_{max} = \sigma^{c}$ that intimately borders on stringer.
The panel intersection area that takes up the compression is determined like a sum of stringers areas and envelope parts with width $b_{кр}$. Then:

$$T_{нес} = 2\sigma^{c}_{гран}\left(F_c + \frac{b_{кр}}{2}\cdot h_o\right)$$

It is possible to be understood under $\sigma^{c}_{гран}$:

- liquid limit;
- the critical stress of stringer elements local buckling;
- the critical stress of stringer general buckling.

In most cases, panel destroying is driven by stringer general buckling. As a matter of fact, plate elements unscrewing don't lead to destroying, but it can start the general buckling even before value $T^{o}_{кр}$ attainment by the load.
Equistability criterion is applied during the design of minimum mass panels:

$$\sigma_{д} \le \sigma^{o}_{кр} \le \sigma^{м}_{кр}.$$

In real standardized profiles: $\sigma^{м}_{кр} \approx 1{,}1\sigma^{o}_{кр}$.

Stiffened cylindrical shell analysis under the axial compression
Riveted structure dry compartment analysis

Loading and structural layout features

Axial compression force and bending moment are determinative loads for axial compartment.

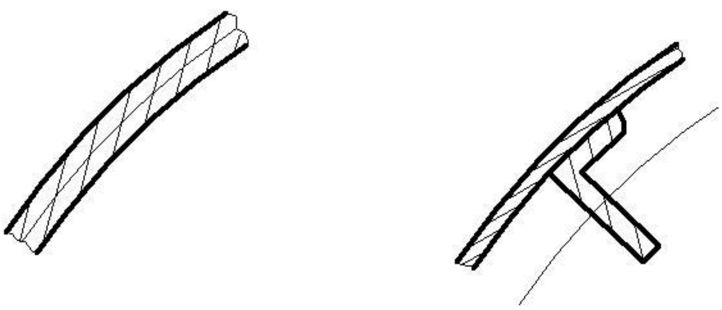
As it known, smooth shells under axial compression force action operate highly inefficiently. If compression critical forces compare with liquid limit, so $\sigma_{кр}/\sigma_{т}$ = 0,1-0,2 will be for smooth unstiffened shells. Small value $\sigma_{кр}/\sigma_{т}$ testify to non-rationality of such shells applying in dry compartments structure. But, anyway, such structures are used in a view of short compartments (for example, during the tanks with dry compartments fitting) as a result of their process – oriented simplicity. Short compartments can be feasible if they don't compose a large part in the aircraft mass balance. Stringer and spar structures are the most widely used in the air force and rocket and space technology.

Structures of such compartments consist of envelope strengthened by longitudinal members like stringers and longerons and also by a transverse frames.

Envelope stability loss between ribs under comparatively low load level is admitted for the most part of such structures. Stringers are principal structural elements in this case.

Stringer structure critical stresses are significantly higher than the ones of equivalent on mass smooth shell. In such structures, it is possible to reach the stress level $\sigma_{кр}/\sigma_{т}$ = 0,4-0,5.

Analysis from structural theory point of view



а) smooth one б) stringer – stiffened one

So, stringer shortening increases massively the $\sigma_{кр}$. The length can be decreased by raising the intermediate frames, then:

$$\sigma_{кр} = \frac{c\pi^2 EI}{F l_{ш}^2}$$

Thus, frames key role is bearings for stringers. But they can only be bearings with sufficient stiffness. So, there has to be presented the specific proportion between $(EI)_с$ and $(EI)_ш$.

As a result of fact, structure behavior will be characterized by the following parameters: $h_о, E_о, E_с, F_с, I_с, E_ш, F_ш, I_ш$ and their number.

Such structure analysis is a complex problem, especially with recesses, envelope immunity loss considering and etc. It can be solved by finite element method, but this is a very labor-consuming deal because approximate approaches are applied in that matter.

Stringers are the principal structure elements and the destroying connects with stringers immunity loss.

The following analytic models are possible to use:

Compartment has the form of sticks kit with the attached envelope, so the functional capability conditions is written in a view of:

$$\sigma^p = \frac{T^p}{F_c^{\Sigma} + F_o^{\Sigma}} \le \sigma^c_{кр.о}\,,\ \sigma^p \le \sigma^c_{кр.о}$$

1. The whole envelope operation is considered in the second analytical model, but the operation with the loads $\sigma^o_{ср}$ and the lifting power of such compartment is considered to be equal to stringers lifting power sum and the joined envelope. Operational capability condition is comfortable to be written down in a form of: $T^p \le T^o_{кр}$, $T^p \le T^м_{кр}$, де $T^o_{кр}$ and $T^м_{кр}$ are envelope lifting powers in a case of general and local stringers immunity loss.

Principal structural element operation

We have analyzed the way of determining the panel lifting power stiffened by stringers approximately before. Let’s distribute this approach for shell lifting power determination stiffened by stringers and frames.

Design fundamental concept is a lifting power of classic structure dry compartment that works in compression is approximately equal to stringers and envelope lifting power sum. Let’s remind once again the necessary formulas shortly. Now, we will consider the simplified version. You should have a look at «Rocket structures integrity» where more exact relations are presented.

Shell analysis on a general immunity

Stringers

Analysis according to bar immunity formulas

Two forms of immunity loss are possible. The first one is general and the second one is local.

General immunity

General immunity critical stresses are defined without the envelope taking into account. Envelope effect is considered a little bit later during shell lifting power determination on the whole.

$$\sigma^{c*}_{кр.о} = \frac{C\pi^2 E_c I_c}{l^2_{ш} F_c}$$

If $\sigma^{c*}_{кр.о} \le \sigma^c_п$, and $\sigma^c_{кр.о} = \sigma^{c*}_{кр.о}$.

If $\sigma^{c*}_{кр.о} > \sigma^c_п$, then $\sigma^c_{кр.о} = \sigma^c_т \left[1 - (1 - \frac{\sigma^c_п}{\sigma^c_т})\sqrt{\frac{\sigma^c_п}{\sigma^{c*}_{кр.о}}}\right]$.

Envelope between stringers

The envelope is considered like a cylindrical panel that is simply depicted on stringers and frames. Formulas for stiffened panel with envelope angularity accounting are used.

Envelope immunity loss is possible.

It's something like:

$$\sigma^о_{кр} = 0{,}1E_о \frac{h_о}{R} + 3{,}6E_о \left(\frac{h_о}{b_о}\right)^2$$

$$\sigma^о_{ср} = \sqrt{\sigma^о_{кр} \cdot \sigma^о_{max}}$$

In the elastic field:

$$\varepsilon_c = \varepsilon_о$$

$$\varepsilon_c = \frac{\sigma^c_{кр}}{E_c};\ \varepsilon_о = \frac{\sigma^о_{max}}{E_о} \to \sigma^о_{max} = \frac{E_о}{E_c} \cdot \sigma^c_{кр.о}$$

Finally:

$T_{нес.о} = n\left[\sigma^c_{кр.о} \cdot F_c + \sigma^о_{ср} b_о h_о\right]$, where n is a number of stringers.

Stringer shell analysis on local immunity

Stringers

Separate plate elements are discriminated in the stringer transversal intersection (booms, webs). Breaking stresses calculation according to compression plates set up is performed for each structure member.

$$\sigma^{c}_{кр.м} = k_з E_c \left(\frac{h_c}{b_c} \right)^2,$$

$$\text{If } \sigma^{c*}_{кр.м} \le \sigma^{c}_{п}, \text{ and } \sigma^{c}_{кр.м} = \sigma^{c*}_{кр.м}.$$

$$\text{If } \sigma^{c*}_{кр.м} > \sigma^{c}_{п}, \text{ then } \sigma^{c}_{кр.м} = \frac{\sigma^{c}_{т}}{A + \sqrt{A^2 - 1}},$$

$$A = 1 + \frac{1}{2} \frac{\sigma^{c}_{п}\sigma^{c}_{т}}{\left(\sigma^{c*}_{кр.м}\right)^2} \left(1 - \frac{\sigma^{c}_{п}}{\sigma^{c}_{т}} \right)^2.$$

h_c and b_c are stringer structural member thickness and width that are under consideration;

K is a coefficient that takes into account grip conditions (long plate sides).

In material factor for the web, it is approximately equals to $k_з$=3,6 and for the boom $k_з$=0,38. More exactly: have a look at «Rocket structures design».

Envelope

Breaking and average demolition stress of envelope section is calculated according to analogy about general immunity calculation.

Difference

$$\sigma^{o}_{ср} = \sqrt{\sigma^{o}_{кр} \cdot \sigma^{o}_{max}}, \quad \sigma^{o}_{max} = \frac{E_o}{E_c} \sigma^{c}_{кр.м}$$

Compression stringer shell lifting power in a case of stringers local immunity loss:

$$T_{нес.м} = n \left[\sum_{S} \left(\sigma^{c}_{кр.м} \cdot b_c h_c \right)_s + \sigma^{o}_{ср} b_o h_o \right]$$

S is a stringer profile element number.

$$T_{нес} = \min\left(T_{нес.о}, T_{нес.м}\right)$$

Checking and design calculation

Non-destroying shell condition

$$T^e f \le T_{нес}$$

Safety factor

$$\eta = \frac{T_{нес}}{T^e f} \ge 1$$

Functional capability conditions are used:

$$T^p \le n\left[\sigma^c_{кр.о}F_c + \sigma^o_{ср}b_o h_o\right]$$

$$T^p \le n\left[\sum_s\left(\sigma^c_{кр.м}b_c h_c\right)+\sigma^o_{ср}b_o h_o\right]$$

$$20 \le \Gamma \le 80.$$

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